



Is this the smallest uncountable set? (Continuum hypothesis, G. Cantor 1900).  
 Cannot be disproved (K. Goedel 1940). Cannot be proved (P. Cohen 1963).

We have nothing.

$\emptyset$

We know that we have nothing.

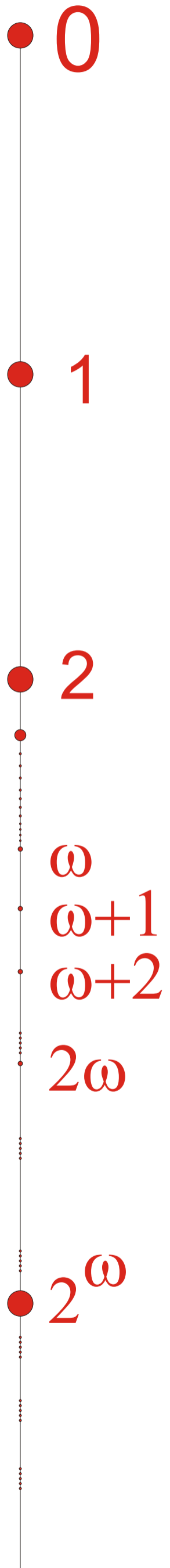
$\{\emptyset\}$

We have nothing and we know that we have nothing.

$\{\emptyset, \{\emptyset\}\}$

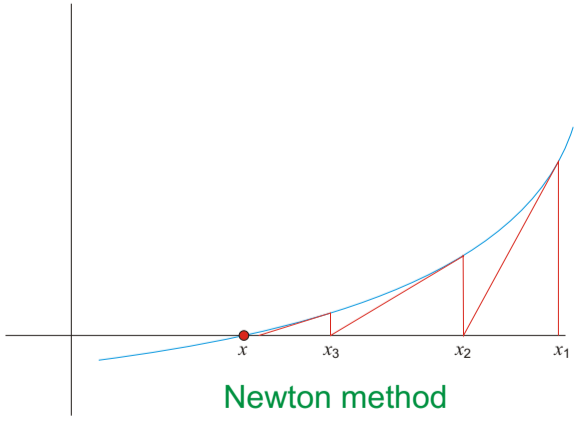
Tidy up !!!

Any set can be well ordered ....

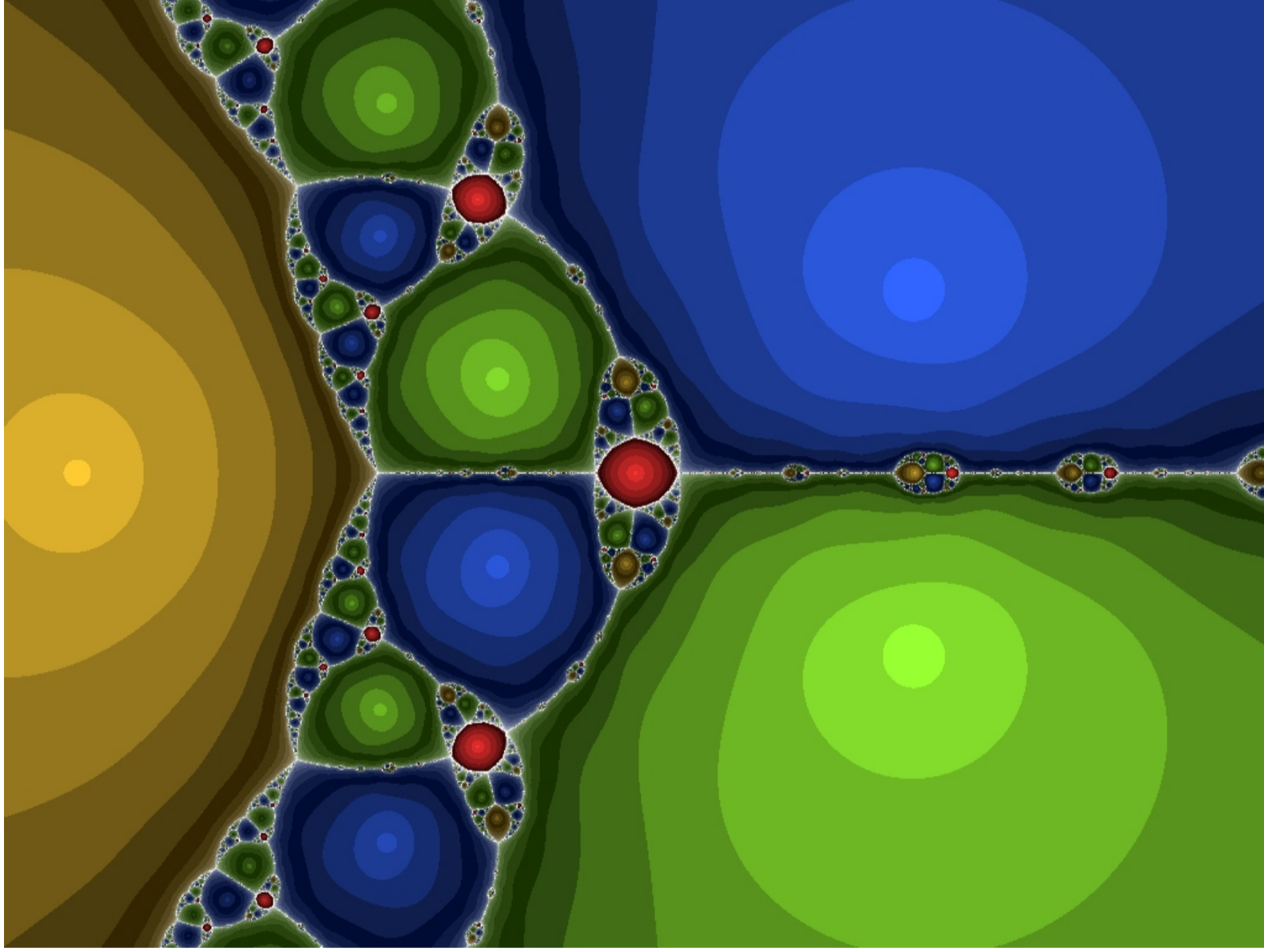
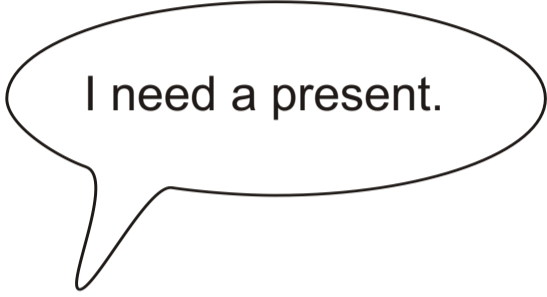
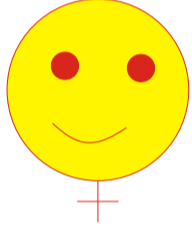


Well-ordering theorem (any set can be well-ordered) is equivalent to the Axiom of Choice (G. Cantor 1897).

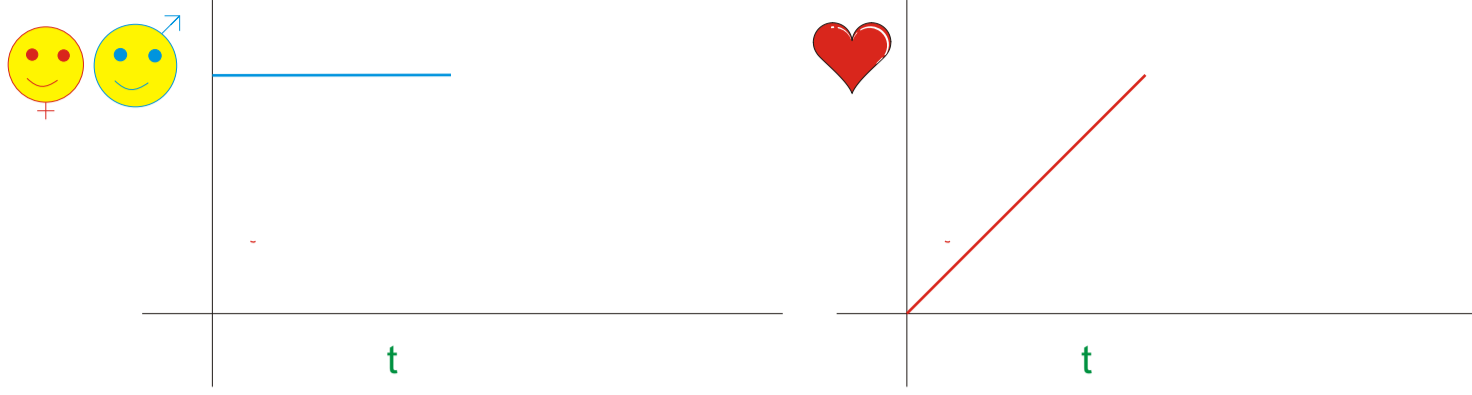
Solution of  $f(z)=z^3 - 2z + 2$  in the range  $[-2,2] \times [-1.5, 1.5]i$  discovered by the Newton method. Orange, blue and green starting points converge to the solutions, red approach both 0 and 1, white points are common boundary (4-corner points).



$$\text{GIFT} = (\text{SMILE})'$$



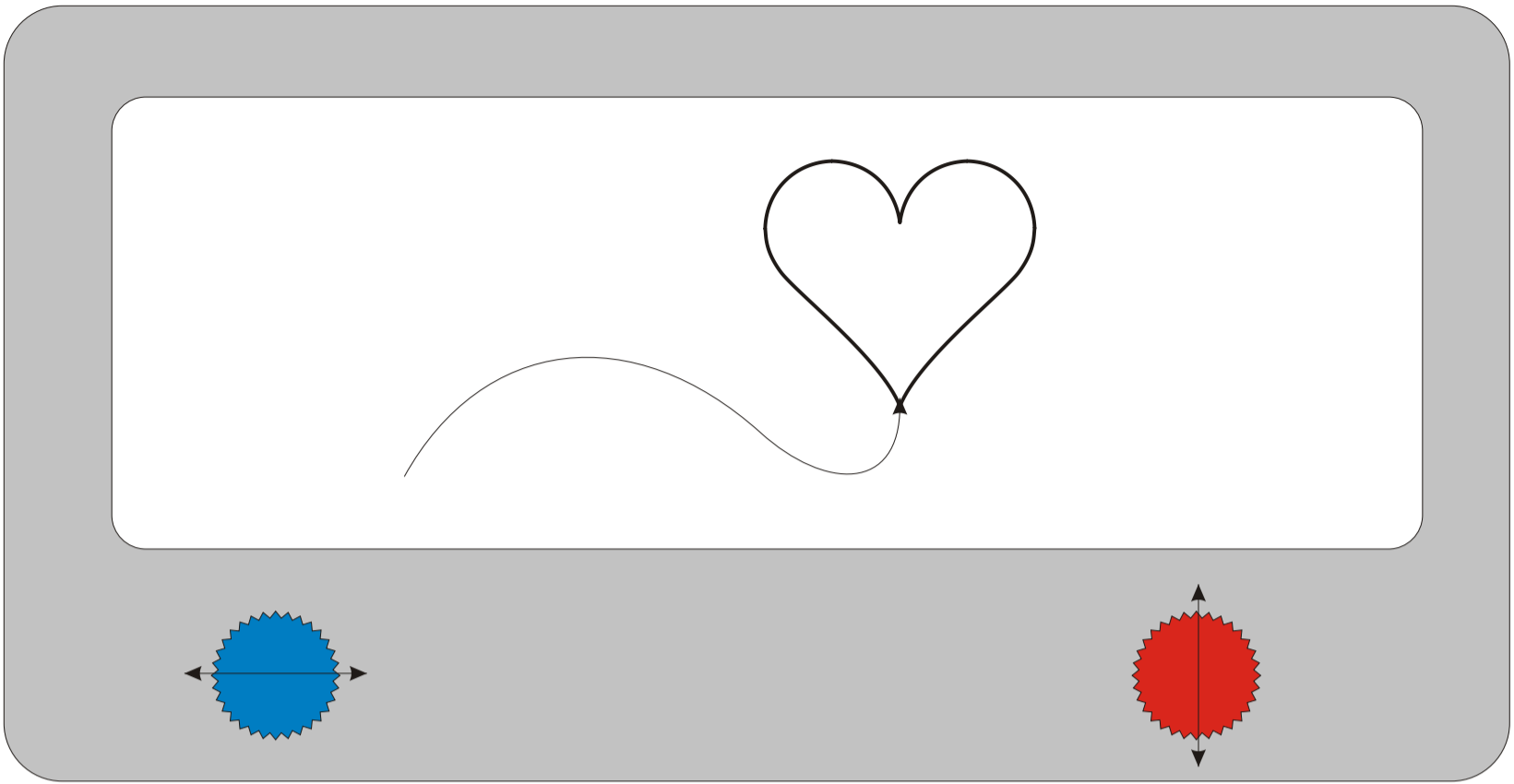
$$\text{SMILE} = (\text{LOVE})'$$



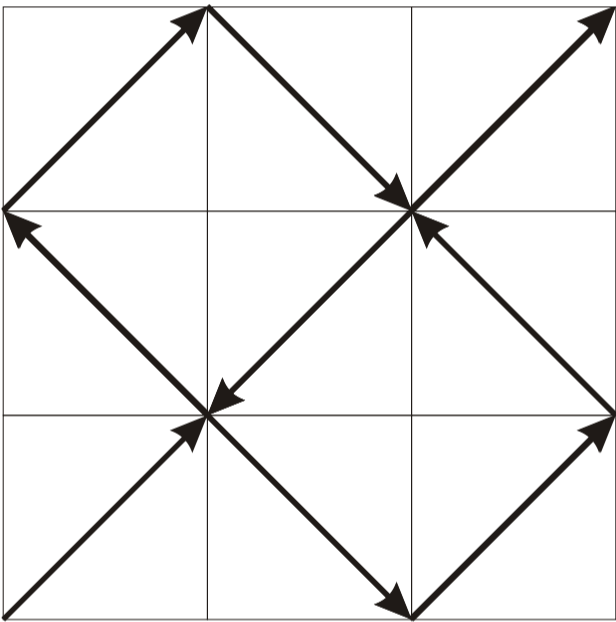
Fundamental theorem of Calculus (I. Newton 1666) states that

$$\int_0^1 \text{SMILE} = \text{LOVE}(1) - \text{LOVE}(0)$$

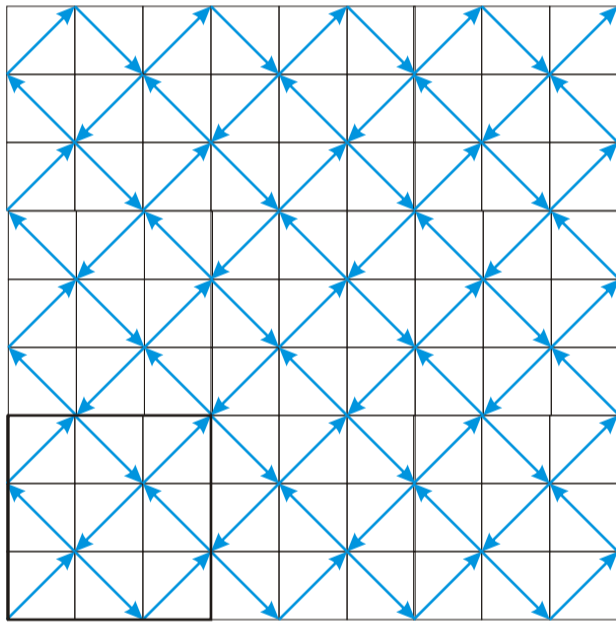
$p_0$



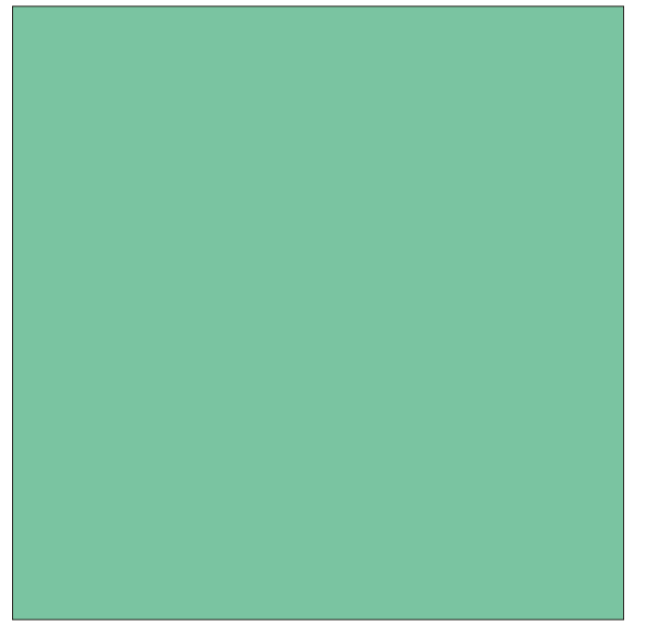
$p_1$



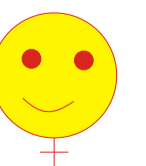
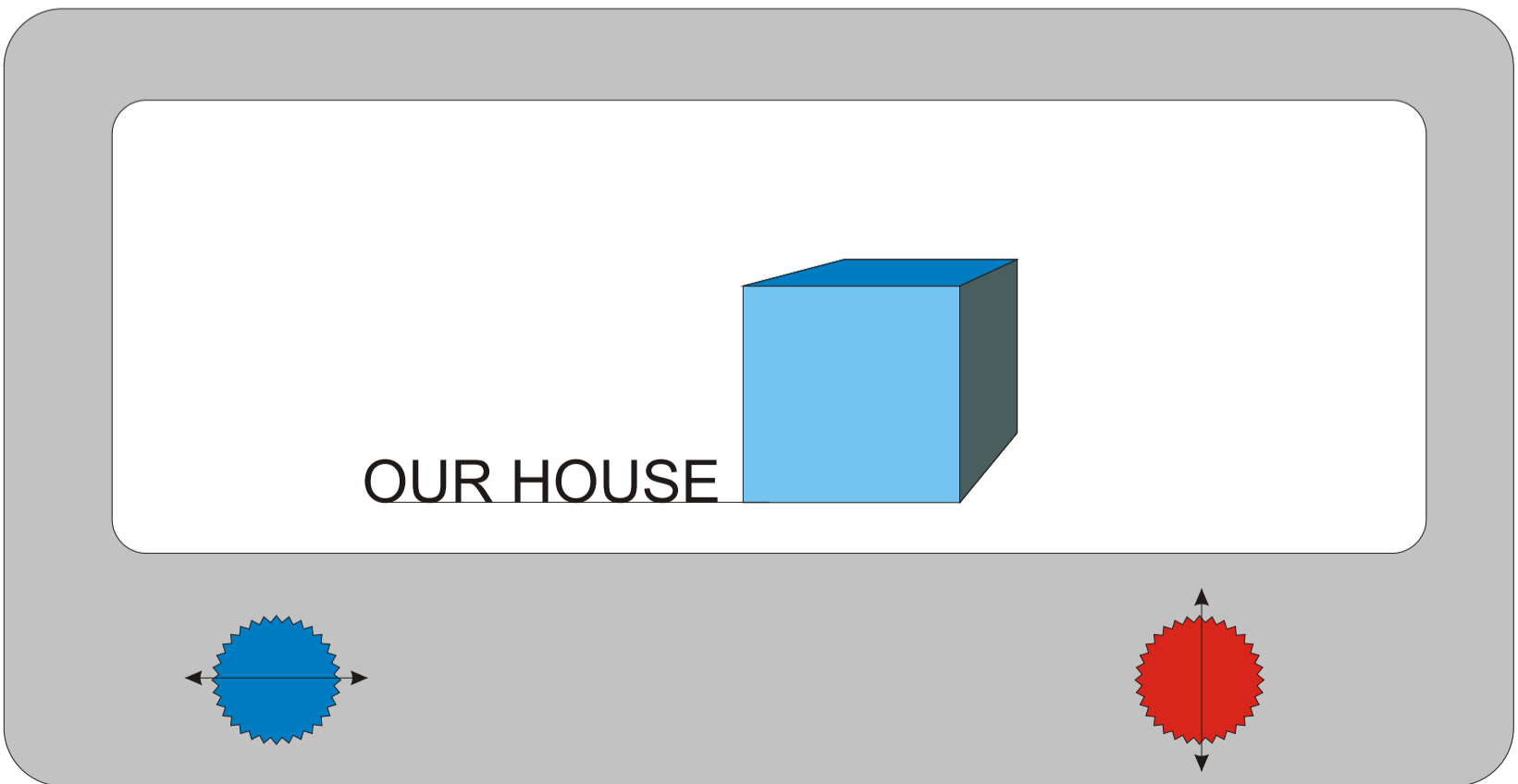
$p_2$



$p_\omega$



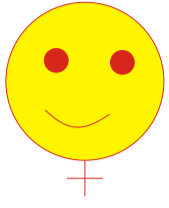
$p_{\omega+1}$



After  $\omega$  steps we obtain from the house the Menger sponge (containing any curve, K. Menger 1926).

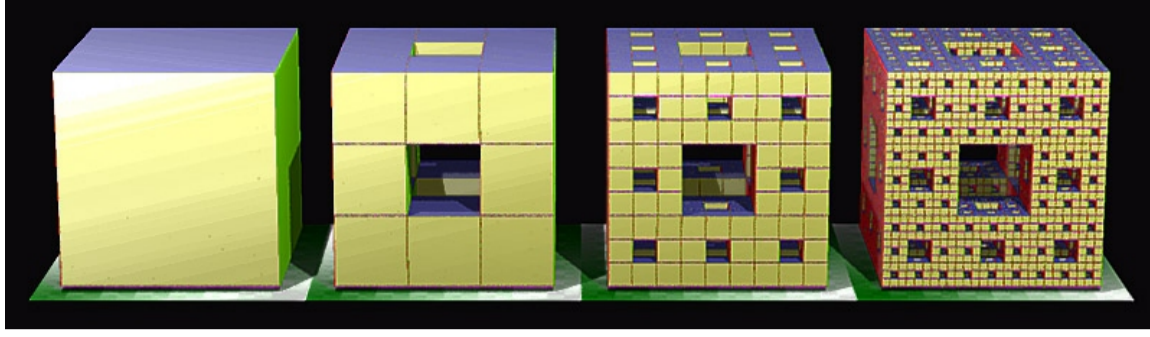
After  $\omega$  steps we obtain from the roof the Sierpinski carpet (containing all plane curves, W. Sierpinski 1916).

After  $\omega$  steps we obtain from the wire the Cantor set (typical compact metric space, G. Cantor 1883).

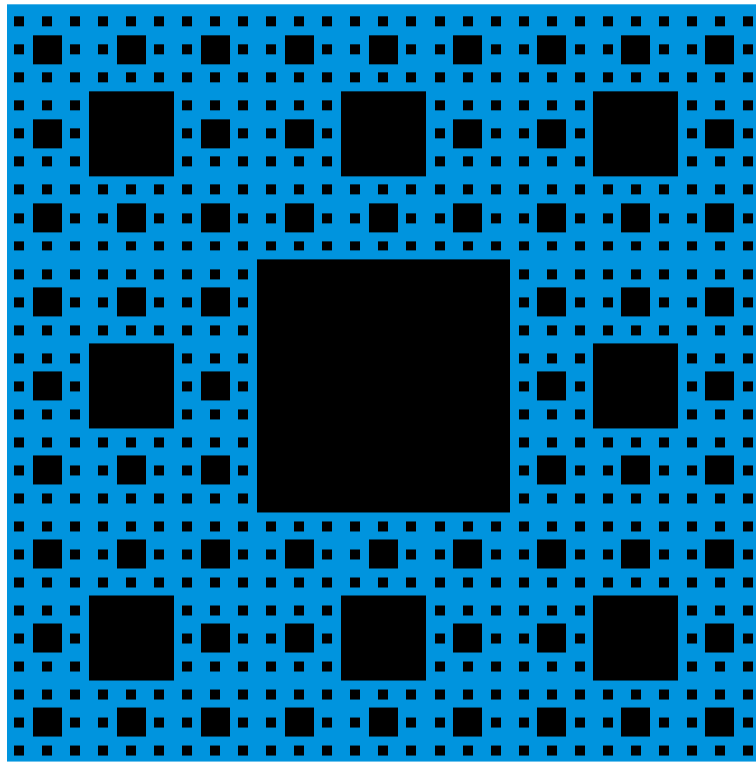


What happened?

Our house was 3 times taxed by  $7/27$ .



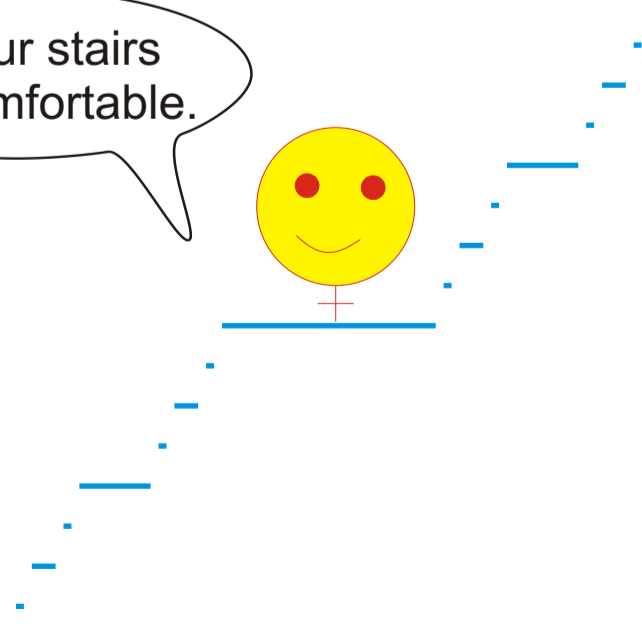
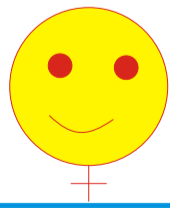
Our roof was 4 times taxed by  $1/9$ .



Our power wire was 4 times taxed by  $1/3$ .



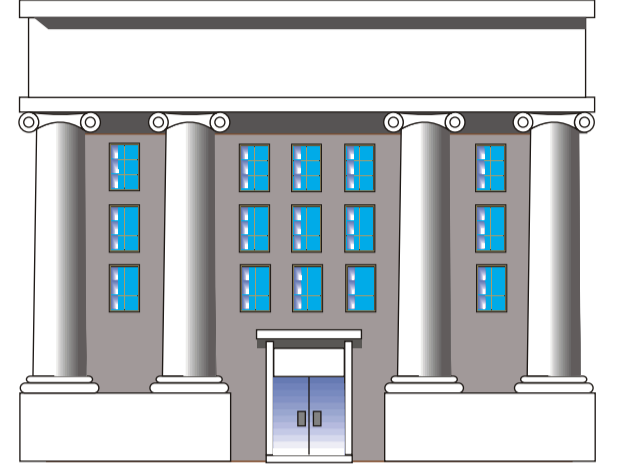
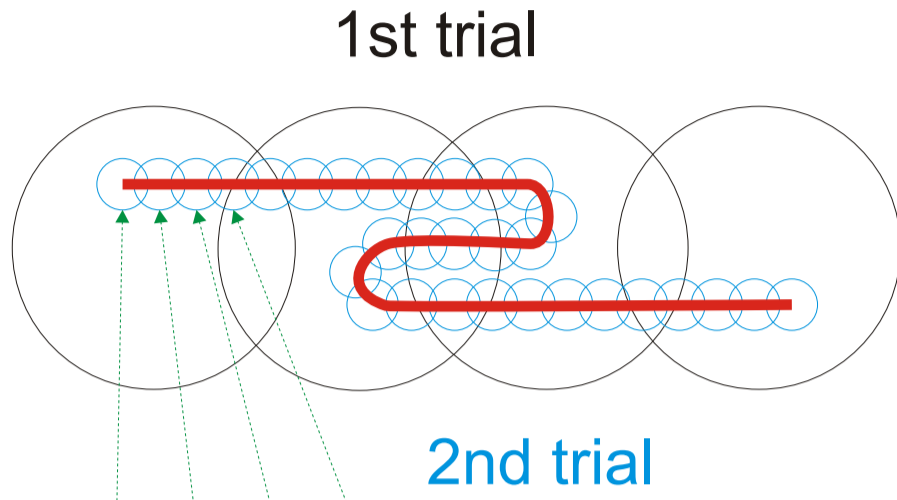
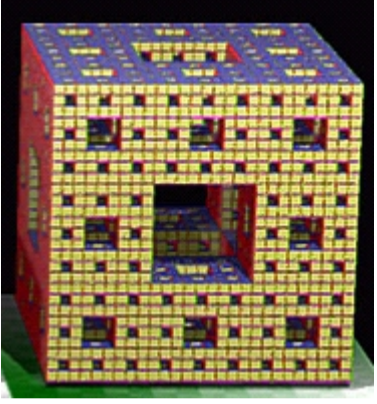
But our stairs are comfortable.



After  $\omega$  trials we obtain the Pseudo-arc (homeomorphic to each its connected closed subset, typical compact connected metric space, B. Knaster 1922).

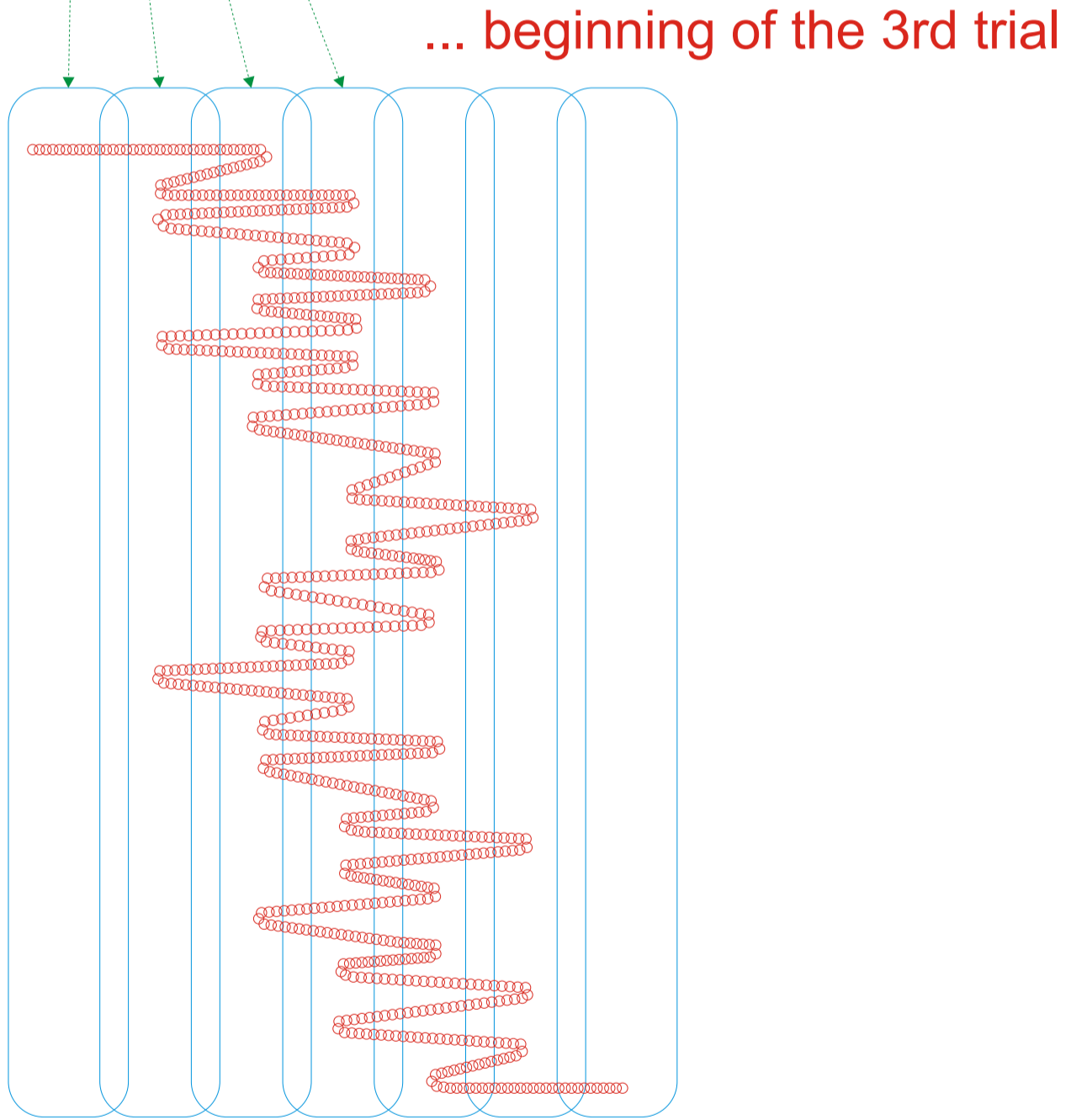


I will go for a tax refund. But I am in doubt if it will go smoothly.

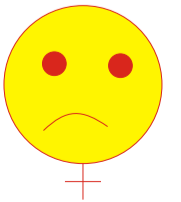


ZOOM

It is crooked.

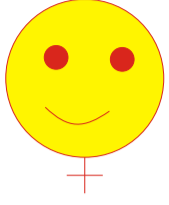


It is arcless & hopeless.

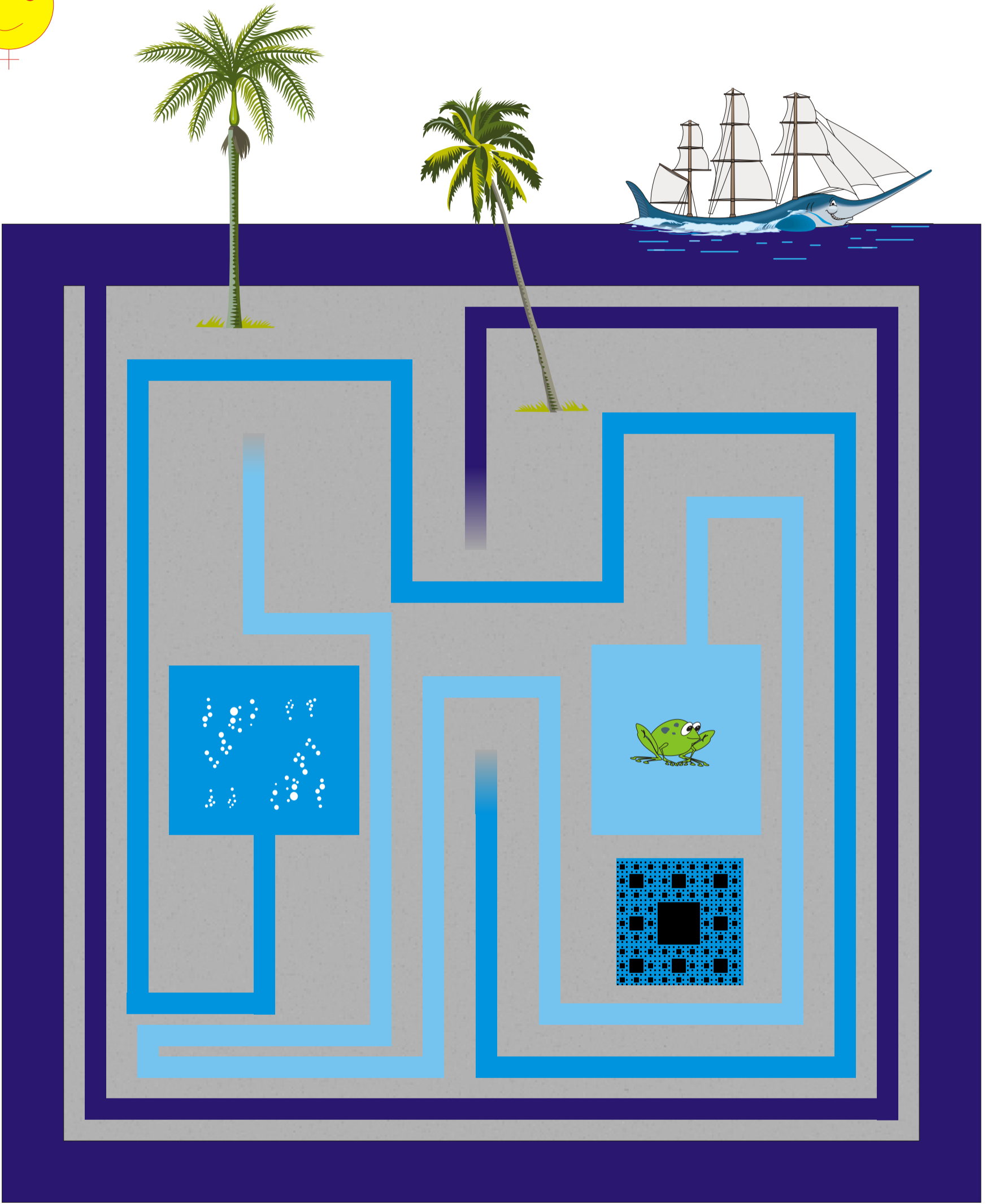




After  $\infty$  digging we obtain 3 disjoint open connected sets of the plane with the same boundary (lakes of Wada, containing salt, sweet and mineral water, K. Yoneyama 1917).



We need sweet, salt and mineral water very very very close. Dig all channels!!!



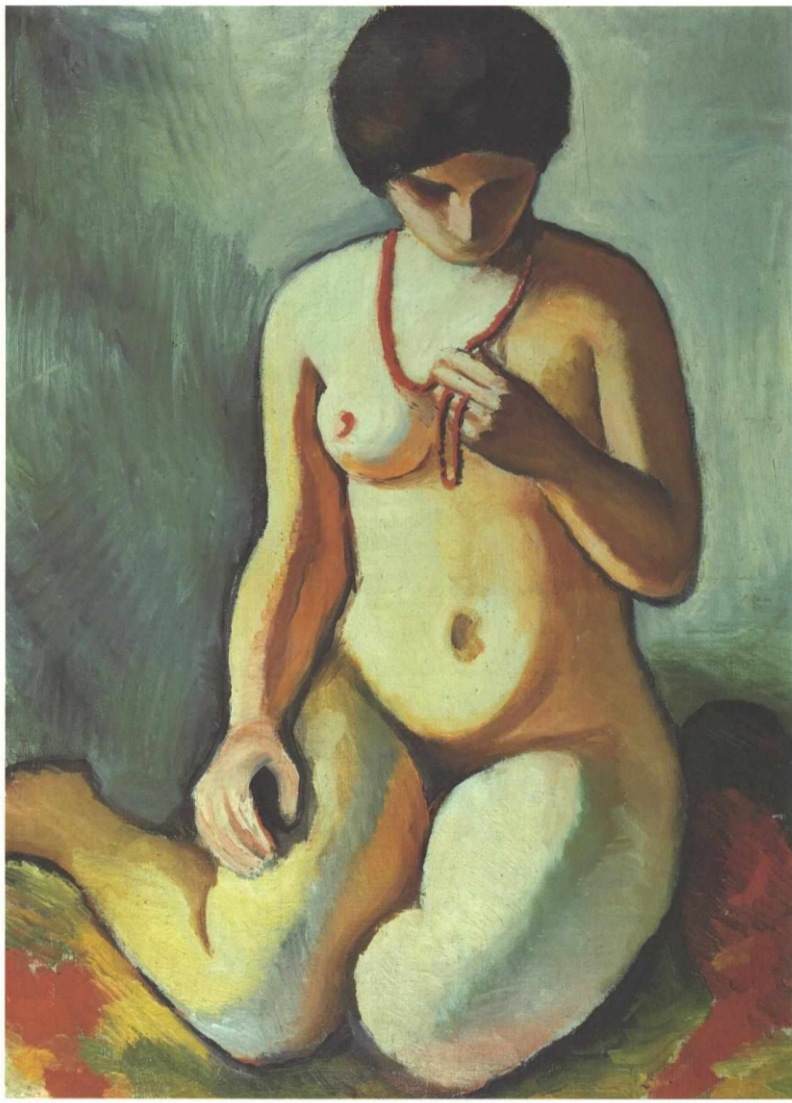
Surely. I can do everything.



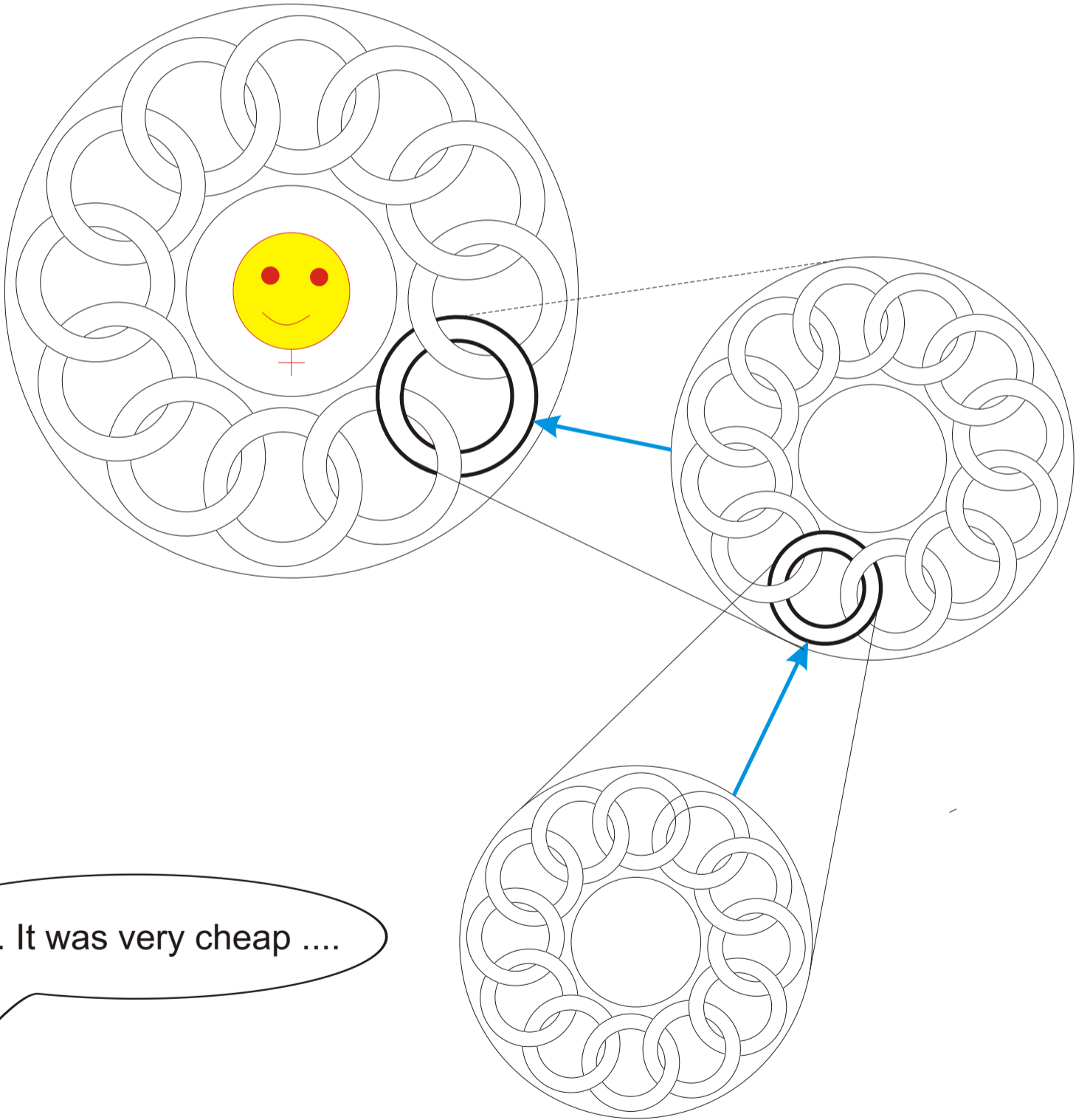
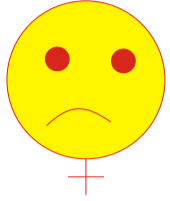
After  $\omega$  carvings we obtain a necklace homeomorphic to the Cantor set embedded in the space in such a way that its complements contain a circle not shinkable to a point (Antoine's necklace, L. Antoine 1921).



Great necklace.



I need also such a necklace.

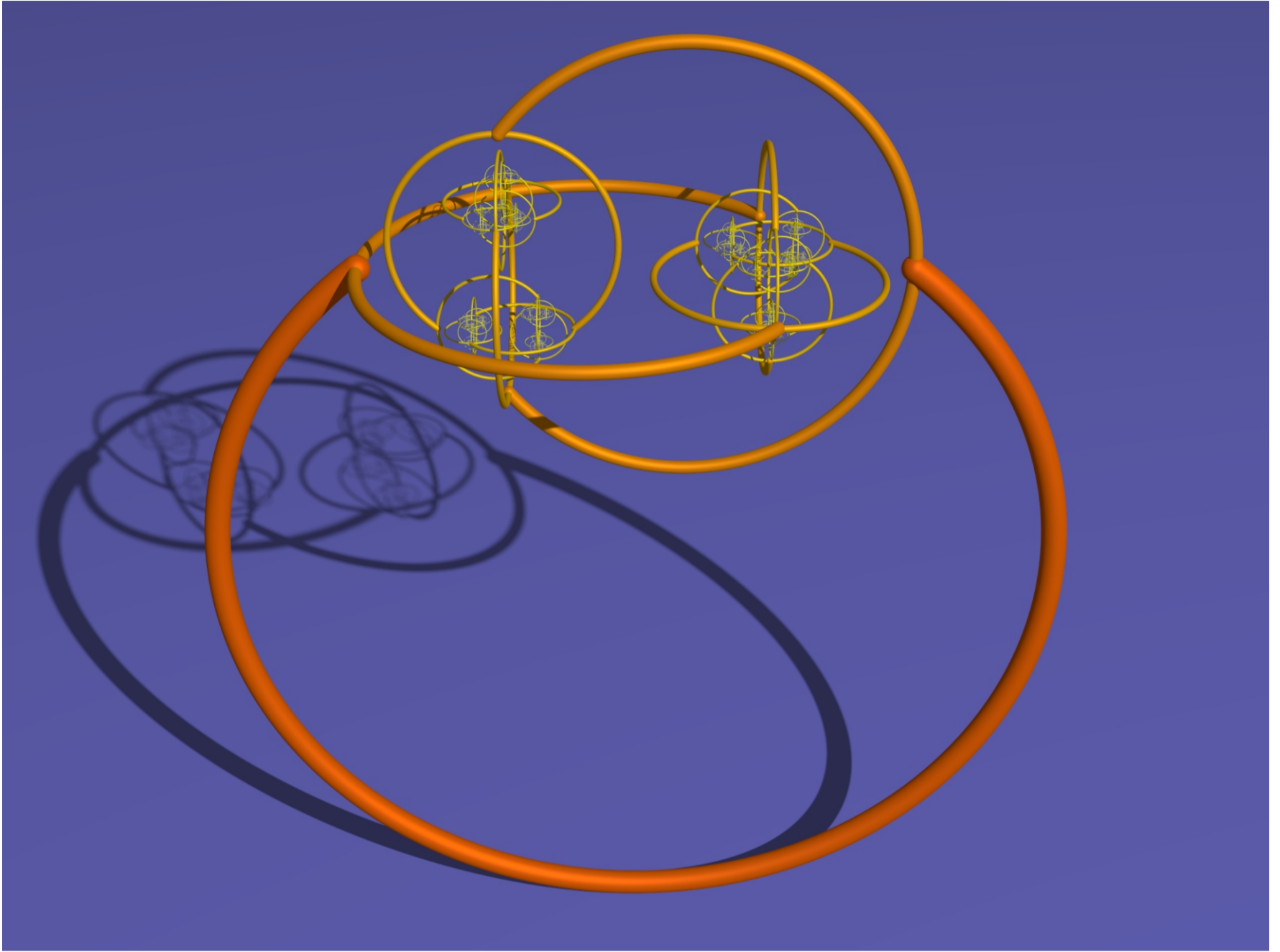


BTW. It was very cheap ....



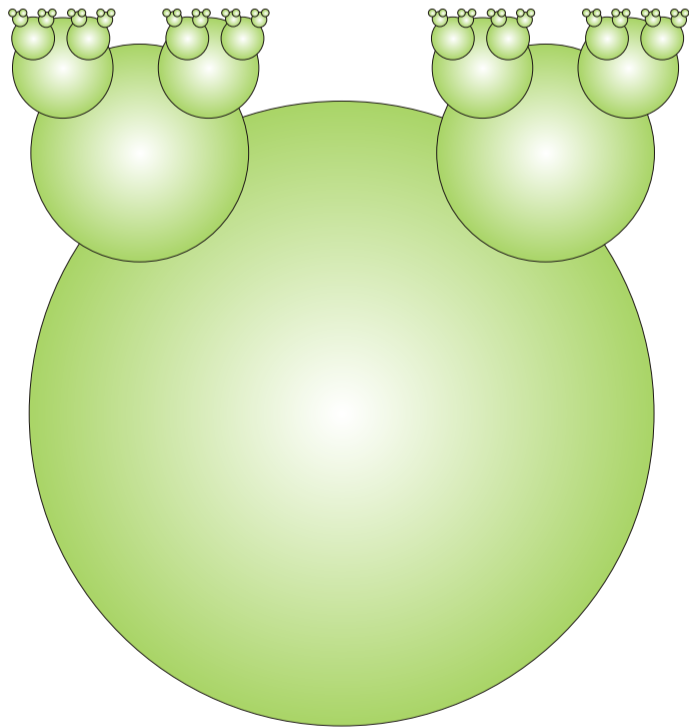


Here is a pendant for your necklace.



The pendant (Alexander horned sphere) is topologically a cube, but its complement contains a circle not shrinkable to a point (Alexander horned sphere, J. W. Alexander 1924).

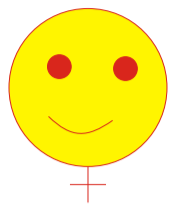
It is topologically a cube.  
It will fall down ...



Keep in mind that  
the pendant is solid, me too.

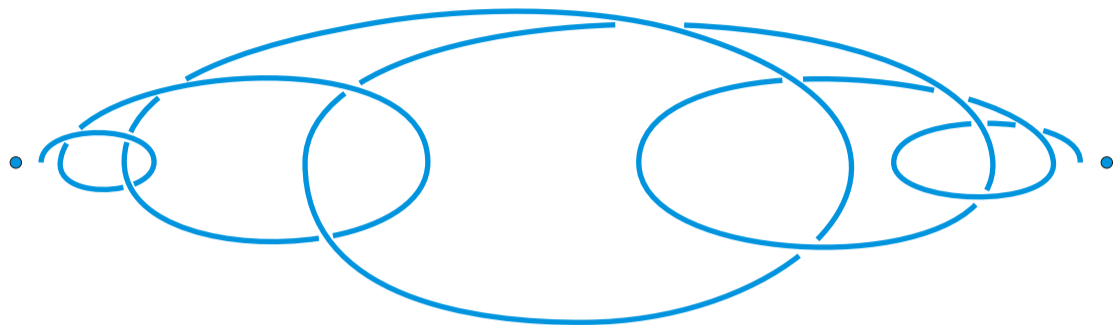


An arc can be embedded in the space in such a way that it's complement contains a circle not shrinkable to a point (wild arc and similarly wild ball and wild burrow, R.H. Fox & E. Artin 1948).

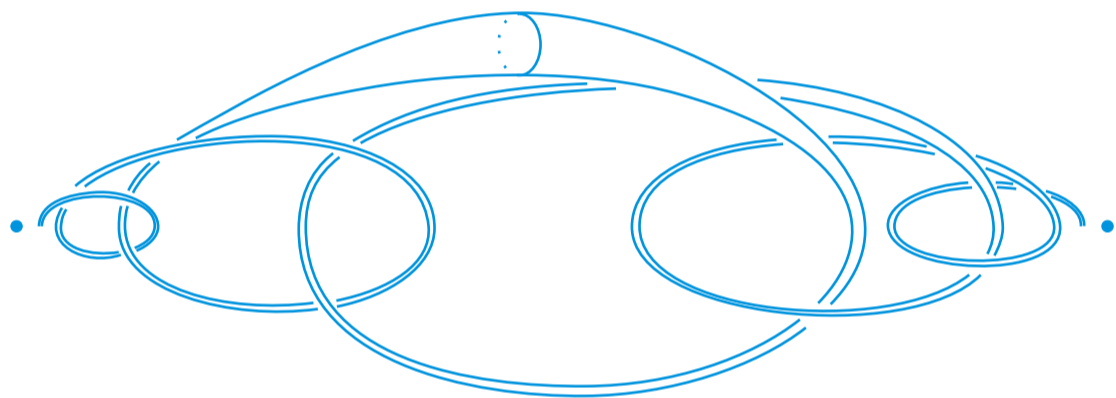


Can you make a pendant from a piece of wire?

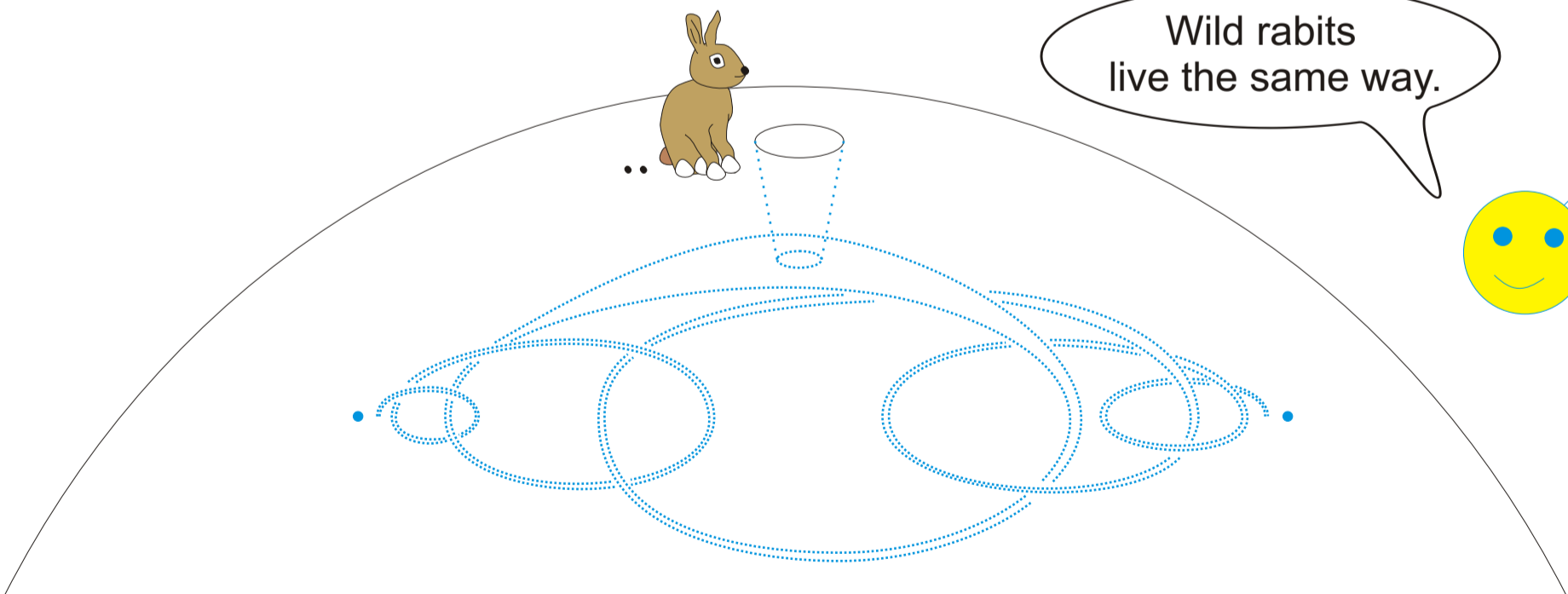
Surely. I can do everything.



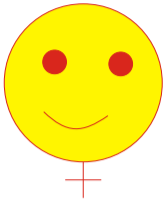
Once I did it with a globe and nobody noticed.



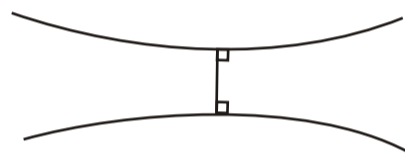
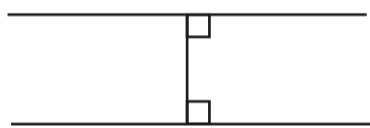
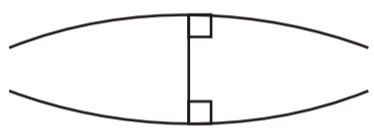
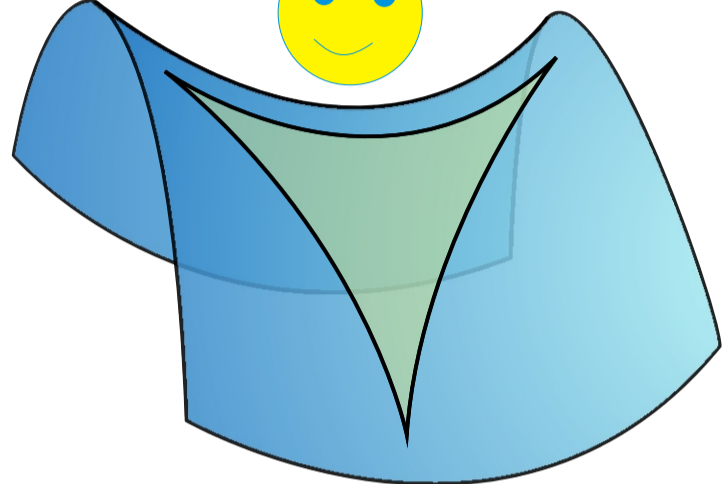
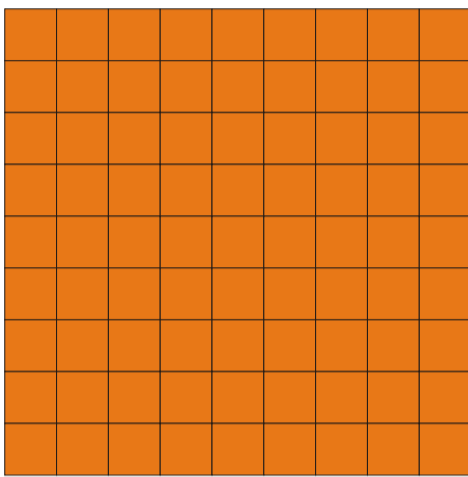
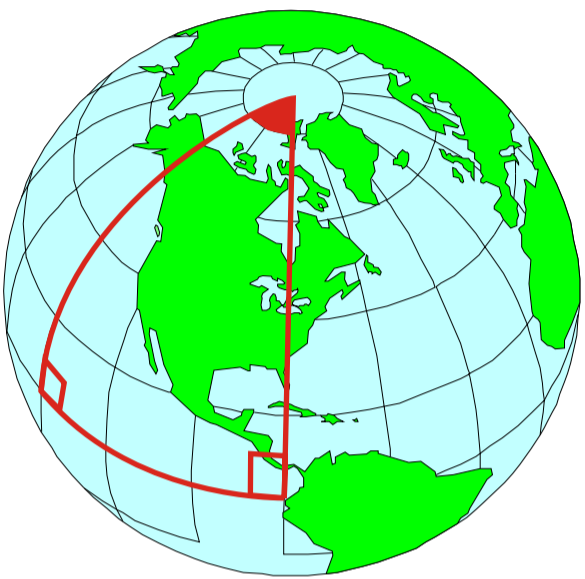
Wild rabbits live the same way.



Do you understand our world?



Surely. But I must check all triangles.



Elliptic

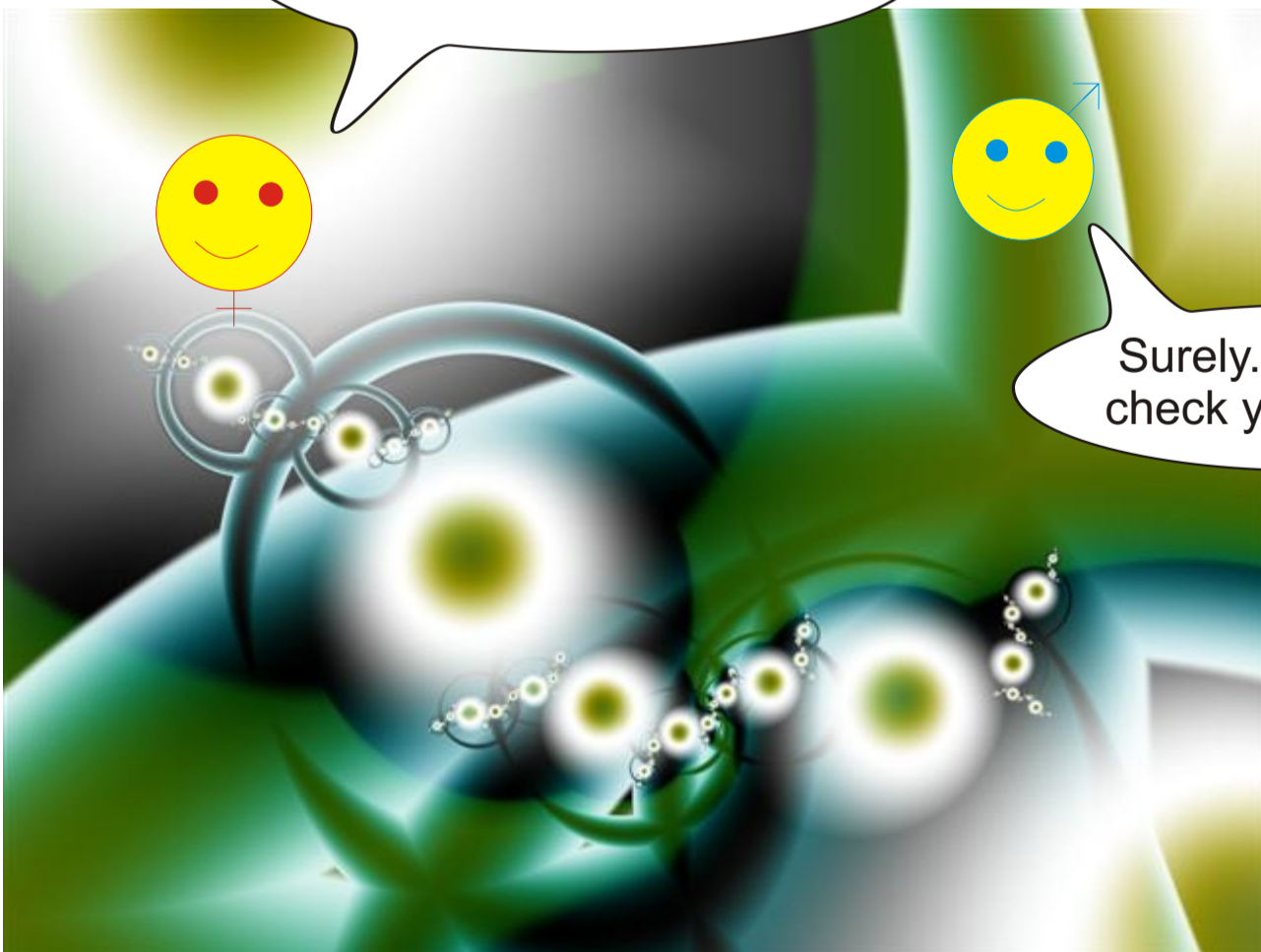
Euclidean

Hyperbolic

Do you understand my world?

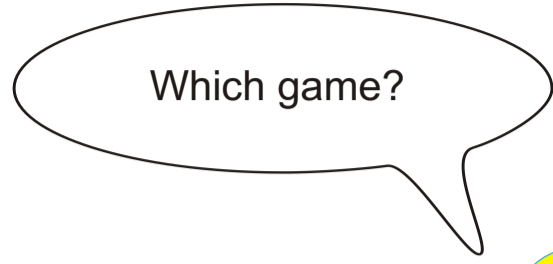
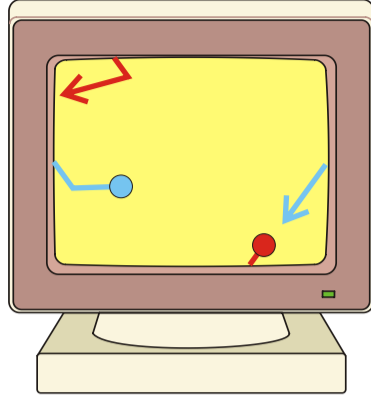
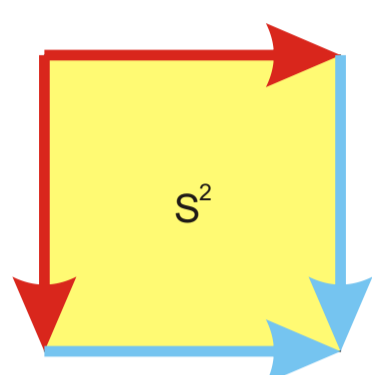
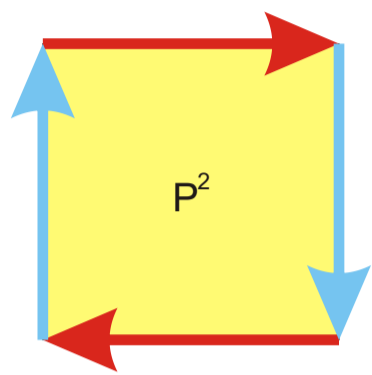
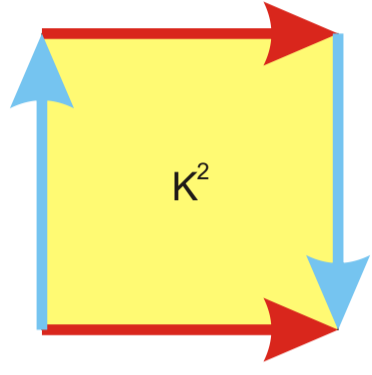
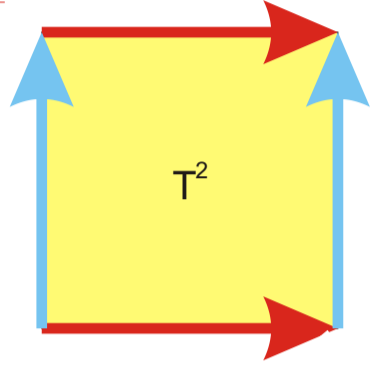
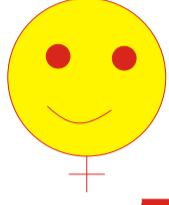
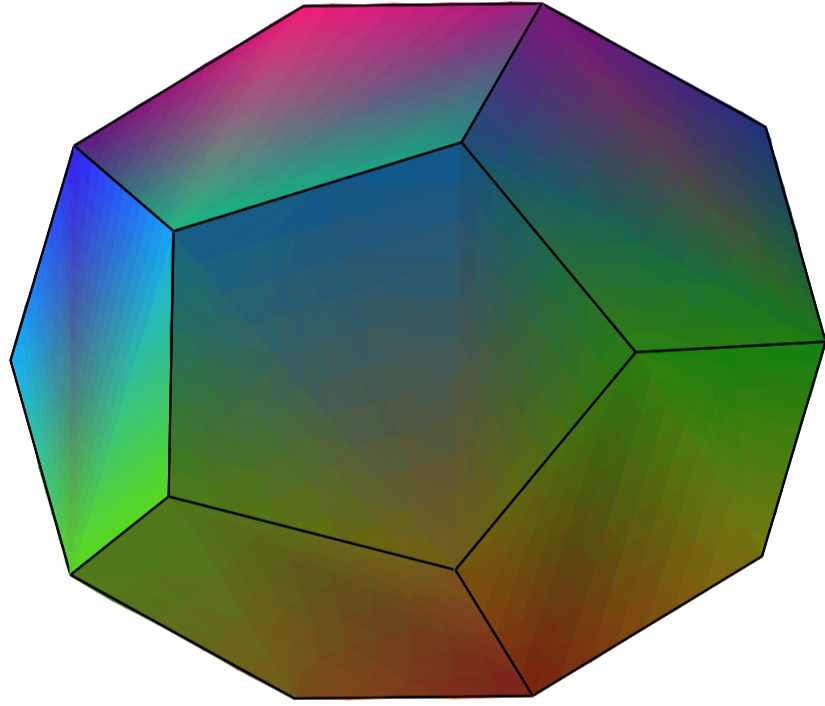
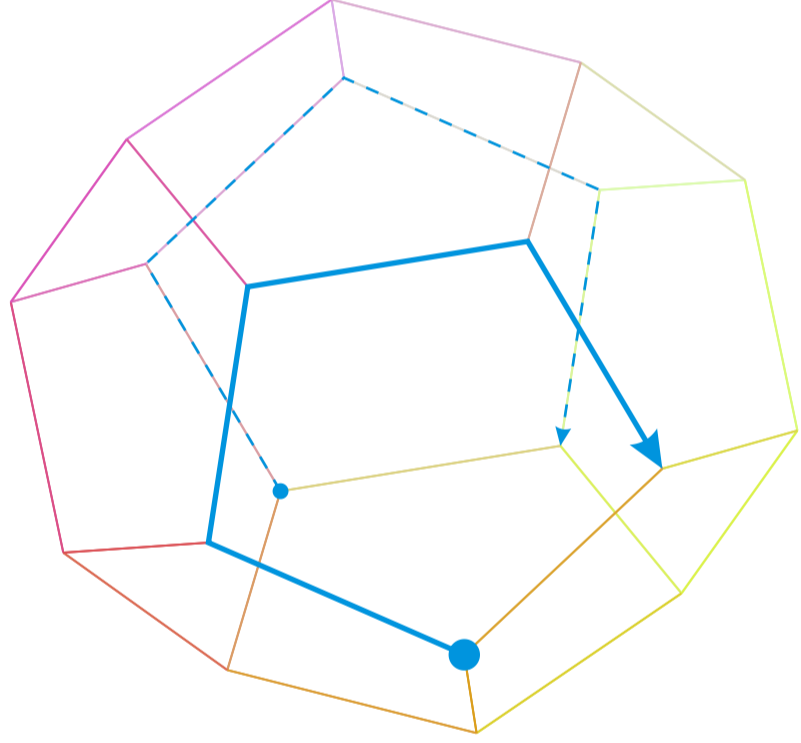
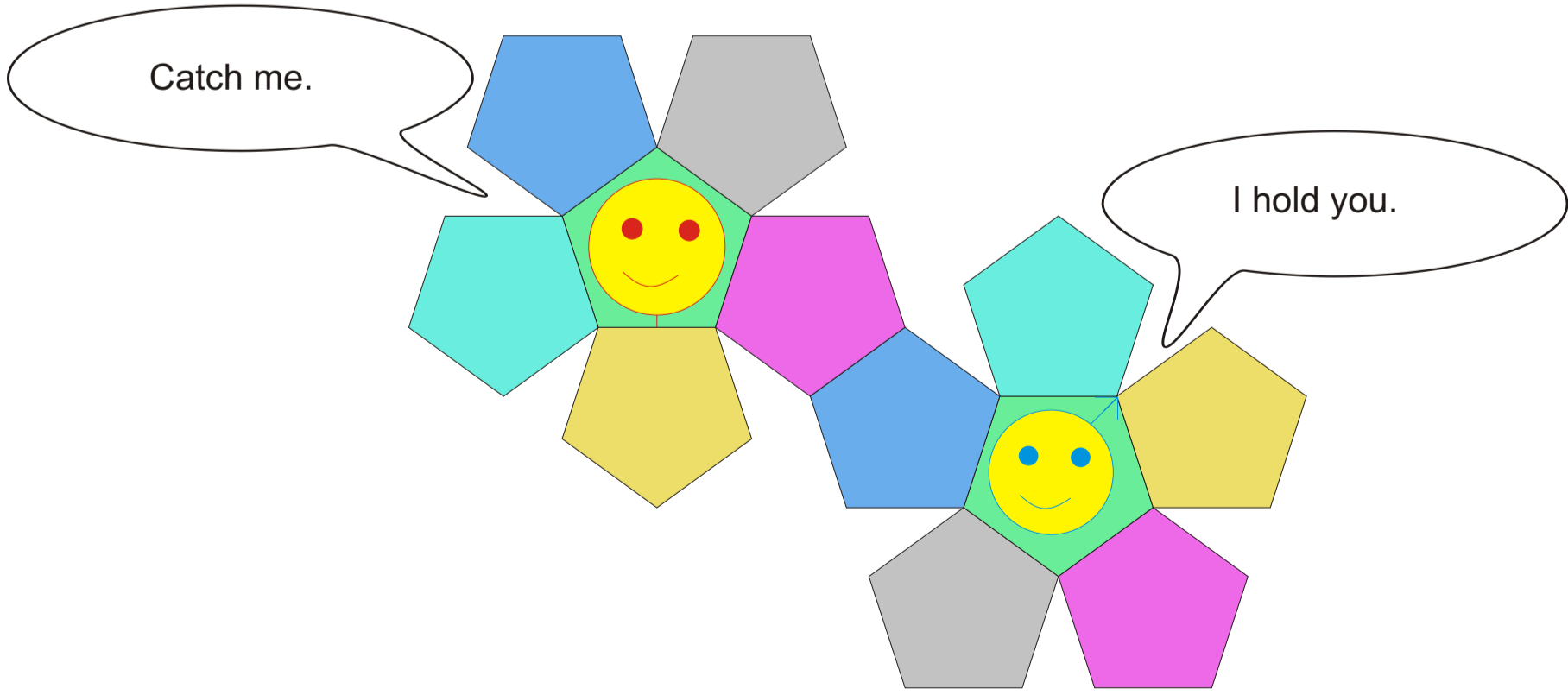


Surely. But I must check your topology.



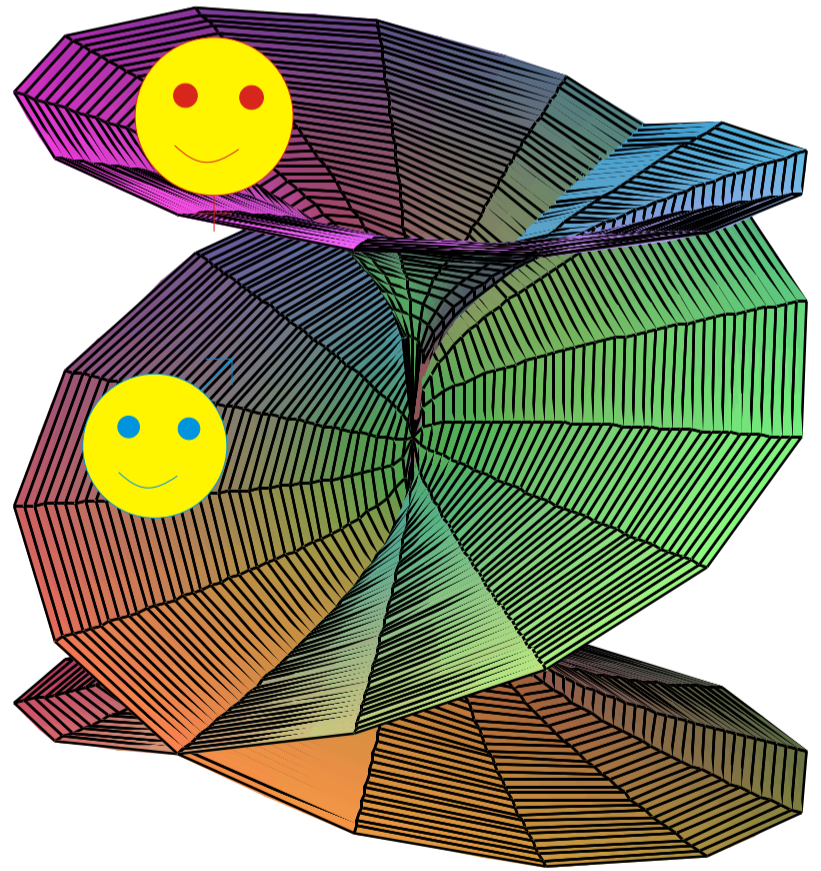
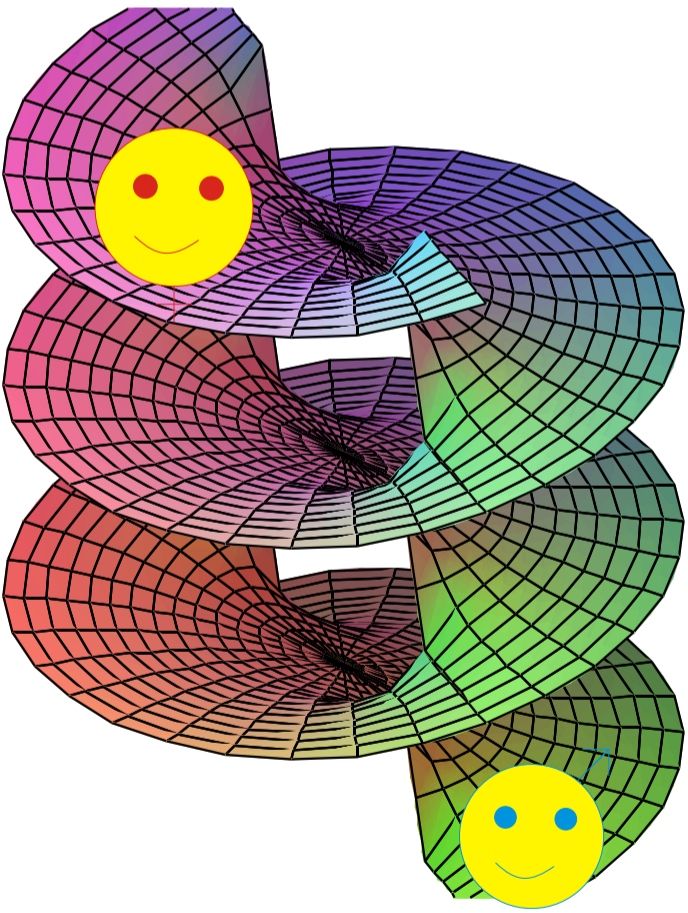
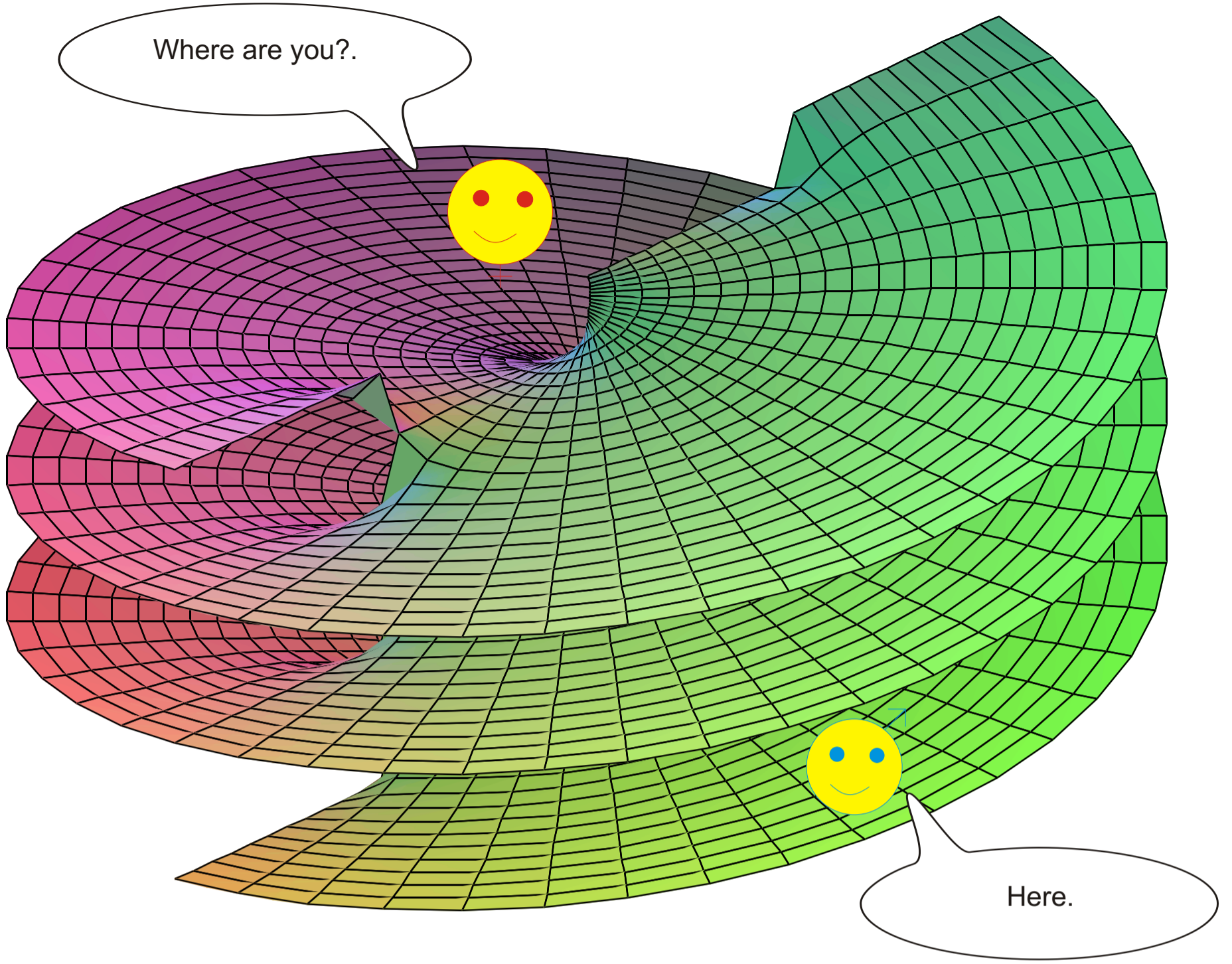
Identifying the opposite sides of the dodecahedron we obtain a strange (but possibly real) 3-D world with torus-like tunnels (Poincaré homology sphere with a fundamental group of order 120, H. Poincaré 1894).

Identifying the opposite sides of the square we obtain four simplest "flat worlds" (torus  $S^2$ , Klein bottle  $K^2$ , projective plane  $P^2$  and sphere  $S^2$ ).





A surface formed gluing three log-like stairs with the same direction (imaginary part of  $\log(x+1) + \log(x-1) + \log(x)$ ).







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Credits: Public domain files

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