

Cardinality theorem

(R. Dedekind 1887, G.Cantor 1895, E.Schroeder 1896, F. Bernstein 1897)

If there exist injective functions $f : A \rightarrow B$ and $g : B \rightarrow A$ between the sets A and B , then there exists a bijective function $h : A \rightarrow B$.

Proof: Consider $B \subset A$ identifying B and its image and denote $C = A \setminus B$. Define $h = f$ on the set $C \cup f(C) \cup f(f(C)) \cup \dots$ and an identity elsewhere.

