

### Cardinality theorem

(R. Dedekind 1887, G.Cantor 1895, E.Schroeder 1896, F. Bernstein 1897)

If there exist injective functions  $f : A \rightarrow B$  and  $g : B \rightarrow A$  between the sets  $A$  and  $B$ , then there exists a bijective function  $h : A \rightarrow B$ .

**Proof:** Consider  $B \subset A$  identifying  $B$  and its image and denote  $C = A \setminus B$ . Define  $h = f$  on the set  $C \cup f(C) \cup f(f(C)) \cup \dots$  and an identity elsewhere.

