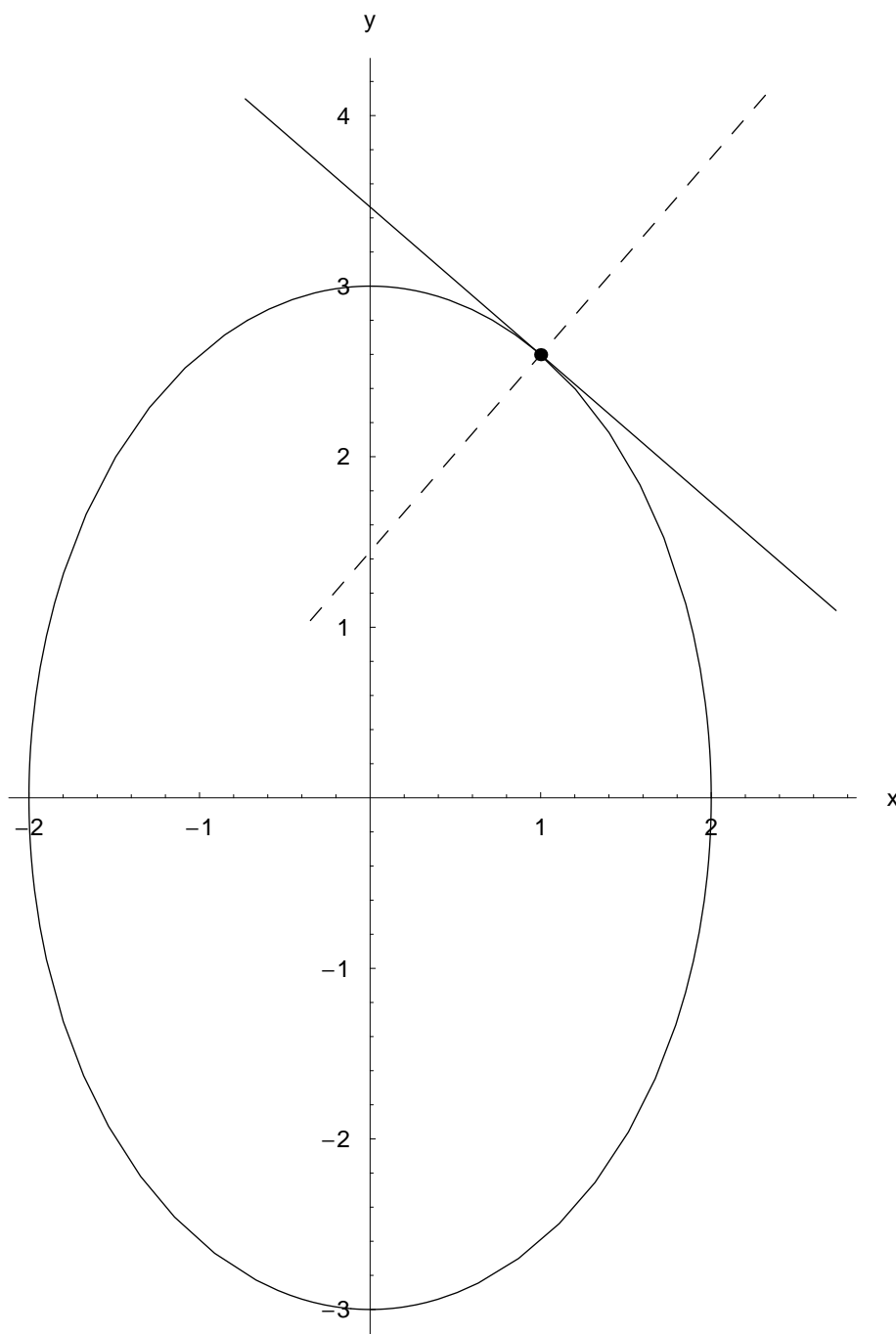


# Obrázky ke kapitole 15

## Cvičení 15.01 na str. 125

Tečna a normála v bodě  $\frac{1}{3}\pi$  k elipse  $f(t) = (2 \cos t, 3 \sin t)$ ,  $t \in \langle 0, 2\pi \rangle$ .

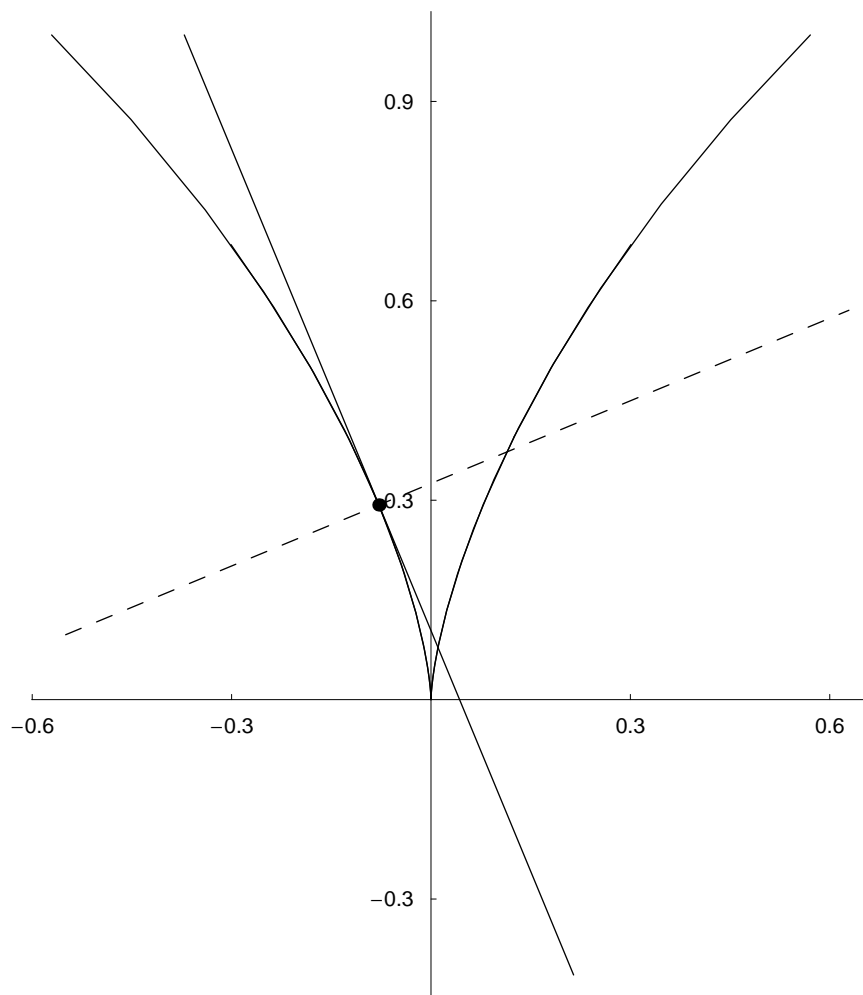
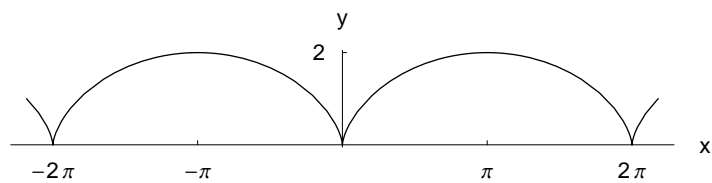


### Cvičení 15.02 na str. 125

Část cykloidy  $f(t) = (t - \sin t, 1 - \cos t)$ ,  $t \in \langle -\frac{5}{2}\pi, \frac{5}{2}\pi \rangle$ .

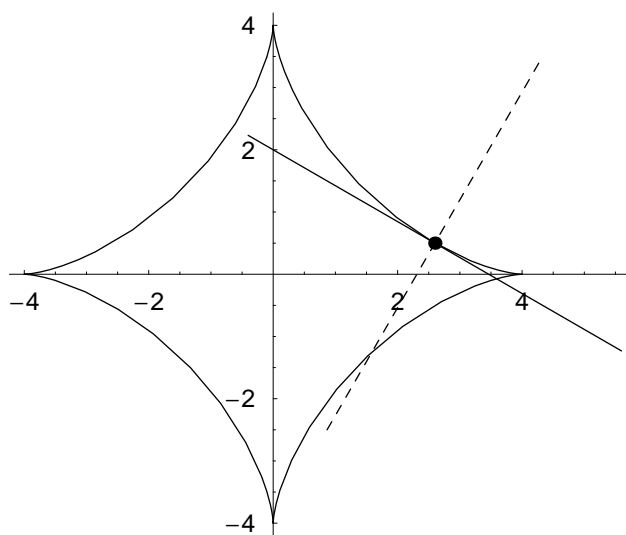
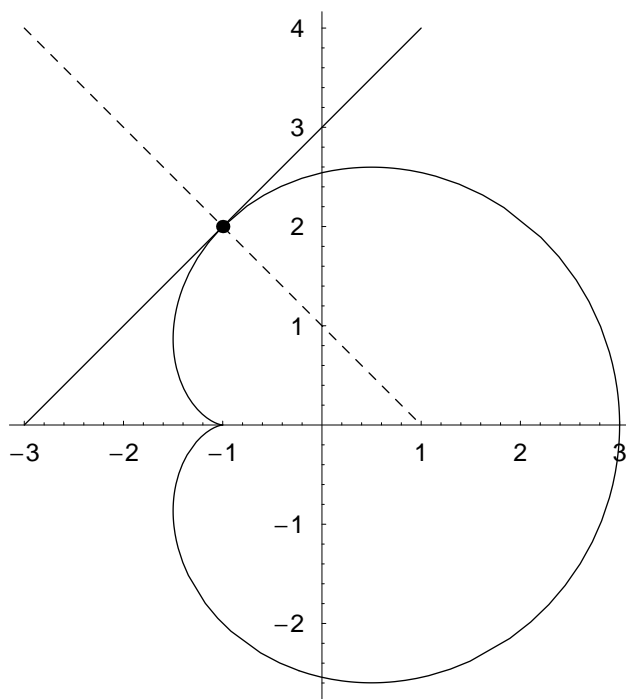
Dole graf restrikce  $f|_{\langle -\frac{1}{2}\pi, \frac{1}{2}\pi \rangle}$  a tečna a normála v bodě  $-\frac{1}{4}\pi$ ;

$$f(-\frac{1}{4}\pi) = (\frac{1}{2}\sqrt{2} - \frac{1}{4}\pi, 1 - \frac{1}{2}\sqrt{2}) \doteq (-0.08, 0.29).$$



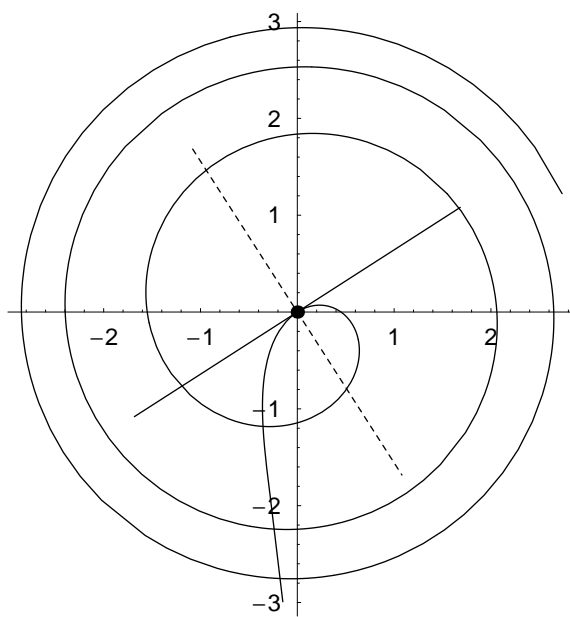
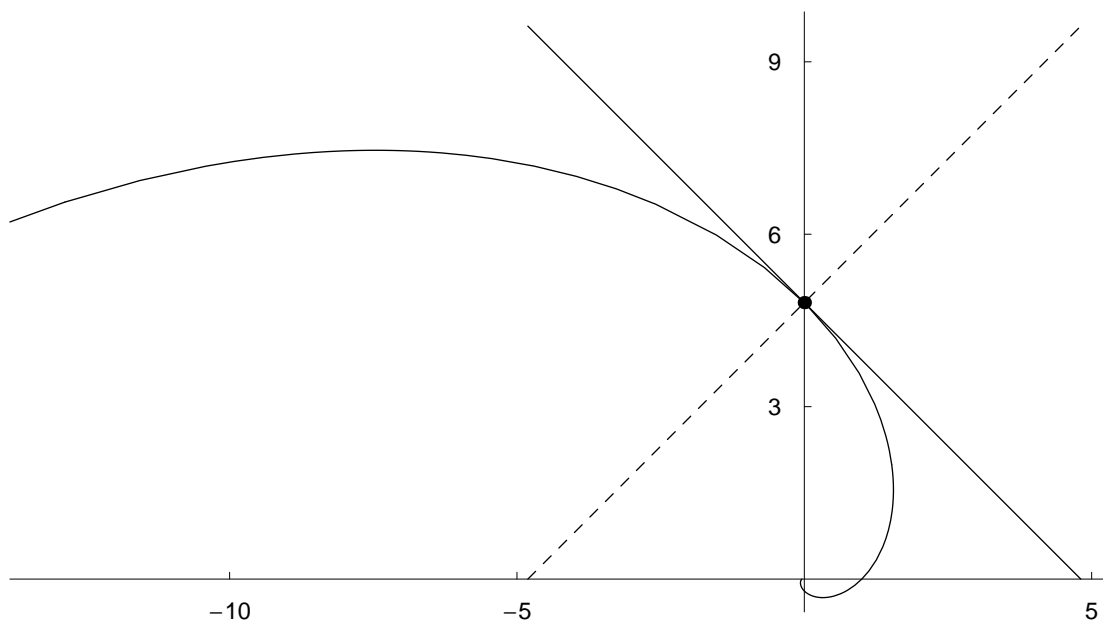
### Cvičení 15.03 a 15.04 na str. 125

Tečna a normála kardioidy  $f(t) = (2 \cos t + \cos 2t, 2 \sin t + \sin 2t)$ ,  $t \in \langle 0, 2\pi \rangle$ , v bodě  $\frac{1}{2}\pi$   
 a astroidy  $f(t) = (3 \cos t + \cos 3t, 3 \sin t - \sin 3t)$ ,  $t \in \langle 0, 2\pi \rangle$ , v bodě  $\frac{1}{6}\pi$ .



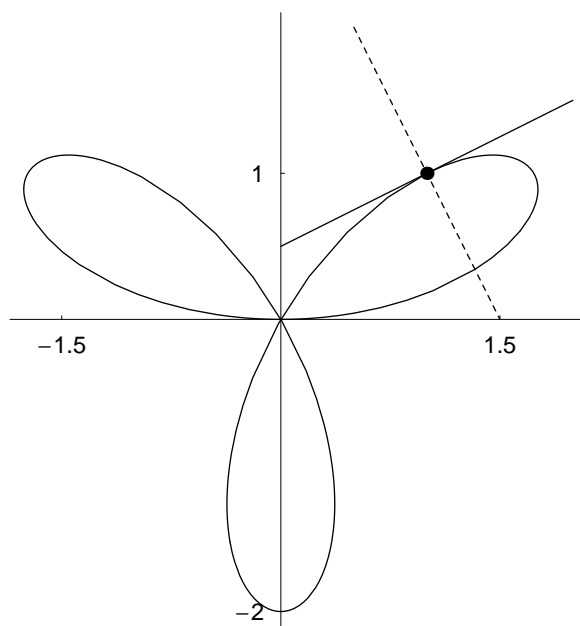
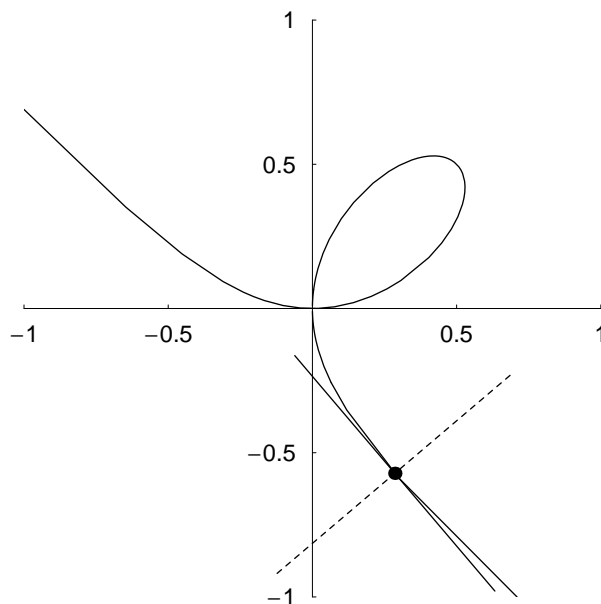
### Cvičení 15.05 a 15.06 na str. 125

Tečna a normála (části) logaritmické spirály  $f(t) = (e^t \cos t, e^t \sin t)$ ,  $t \in \langle -\pi, \pi \rangle$ , v bodě  $\frac{1}{2}\pi$   
 a křivky  $f(t) = (\lg t \sin t, \lg t \cos t)$ ,  $t \in \langle 1/20, 20 \rangle$ , v bodě 1.



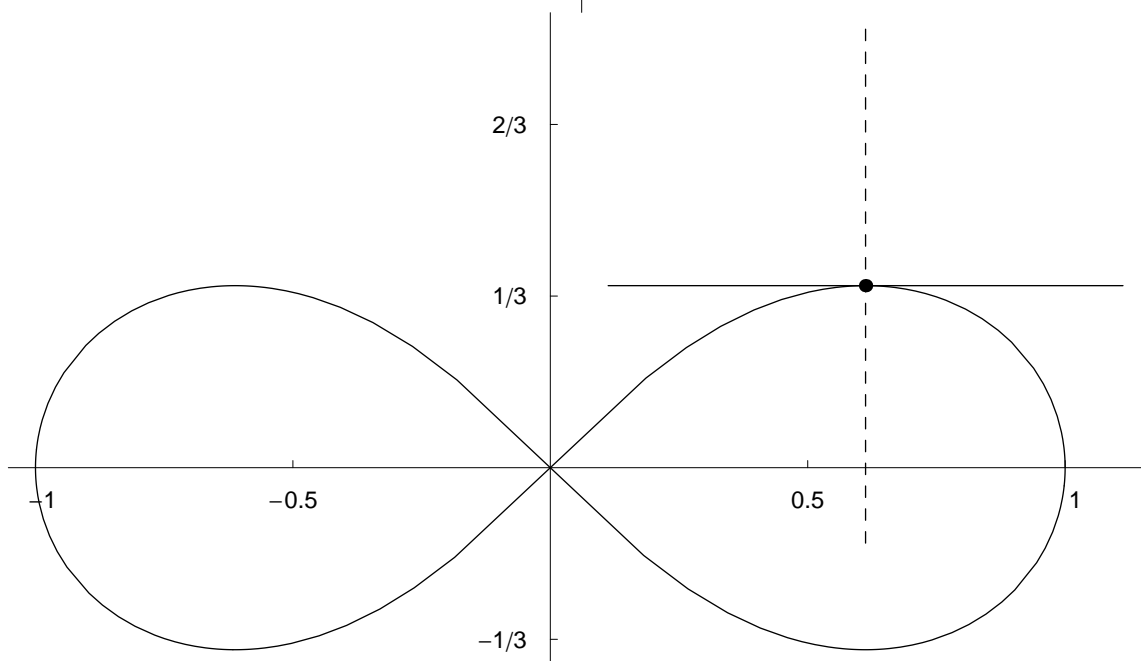
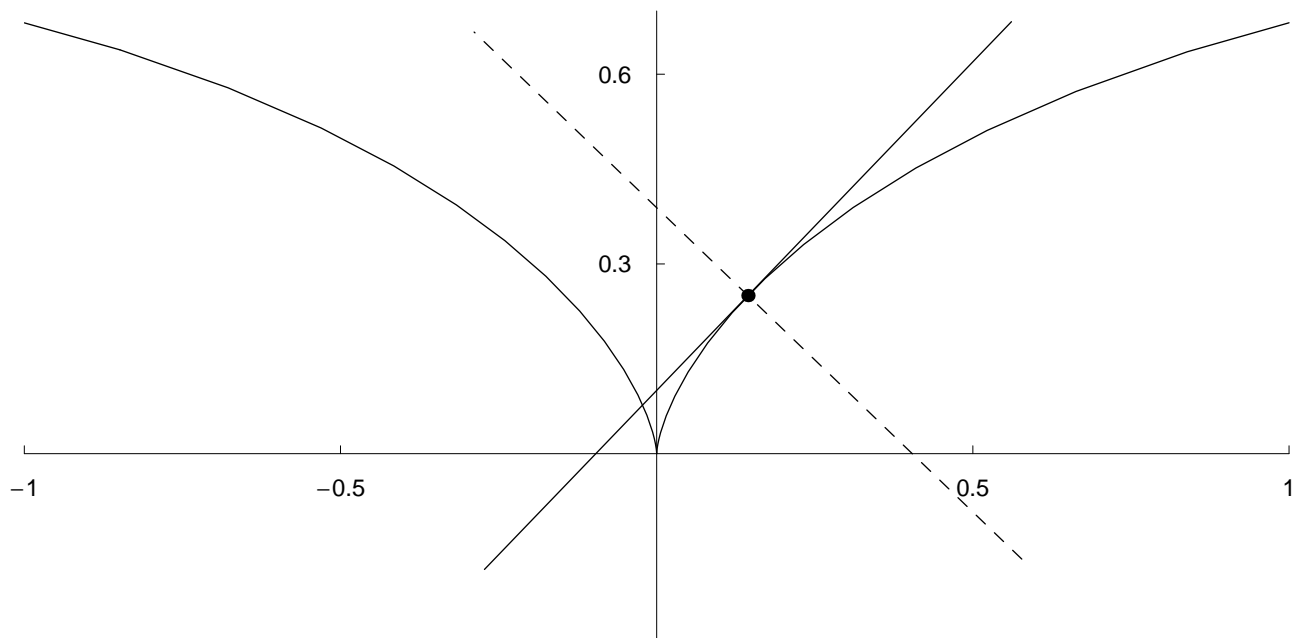
### Cvičení 15.07 a 15.08 na str. 125

Tečna a normála Descartesova listu  $f(t) = (t/(t^3 + 1), t^2/(t^3 + 1))$ ,  $t \in \langle -30, 30 \rangle$ ,  $t \neq -1$ ,  
v bodě  $-2$  a trojlístku  $f(t) = (\cos t + \sin 2t, \sin t + \cos 2t)$ ,  $t \in \langle 0, 2\pi \rangle$ , v bodě  $0$ .



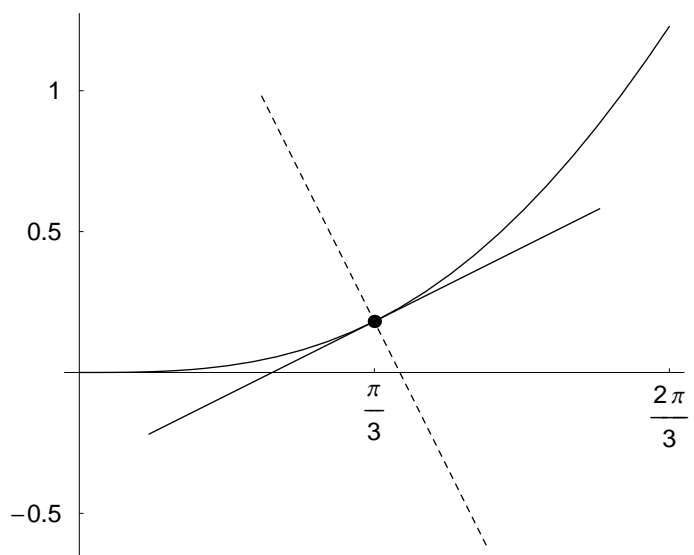
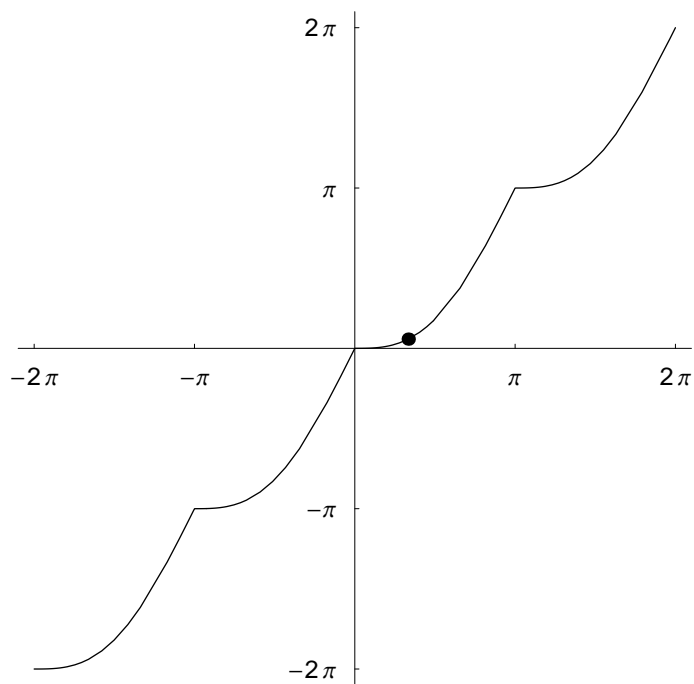
### Cvičení 15.09 a 15.11 na str. 125

Tečna a normála Dioklovy kissoidy  $f(t) = (\cos^3 t / \sin t, \cos^2 t)$ ,  $t \in (-\frac{1}{2}\pi, -0.1) \cup (0.1, \frac{1}{2}\pi)$ , v bodě  $\frac{1}{3}\pi$   
 a lemniskaty  $f(t) = \sqrt{\cos 2t} (\cos t, \sin t)$ ,  $t \in \langle -\frac{1}{4}\pi, \frac{1}{4}\pi \rangle \cup \langle -\frac{3}{4}\pi, \frac{5}{4}\pi \rangle$ , v bodě  $\frac{1}{6}\pi$ .



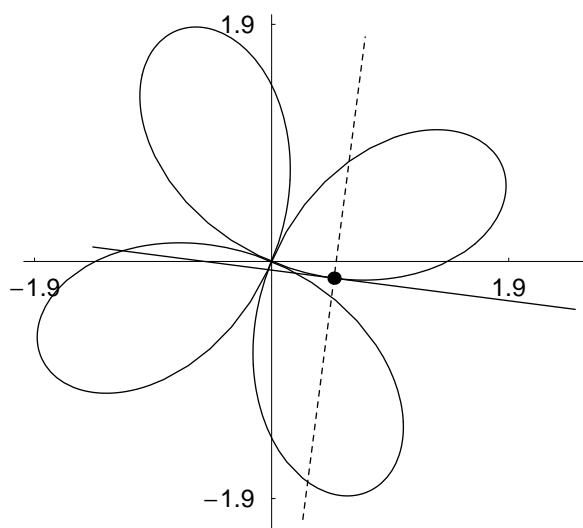
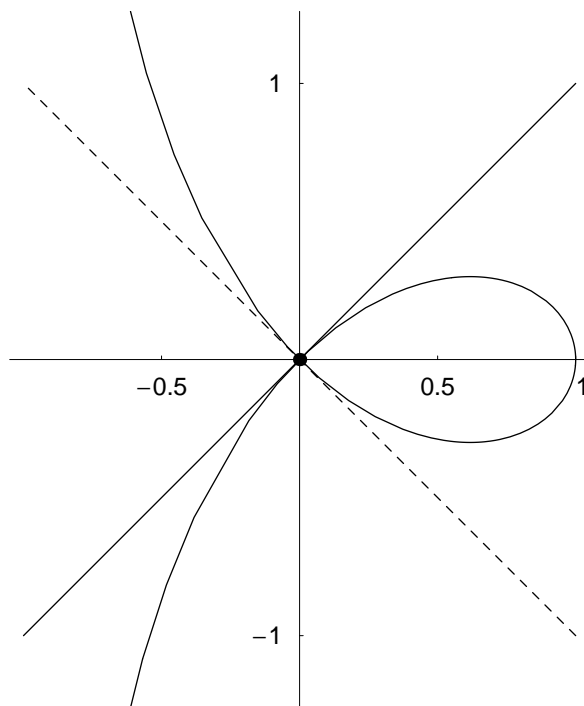
### Cvičení 15.10 na str. 125

Tečna a normála křivky  $f(t) = (t, t - |\sin t|)$ ,  $t \in \langle -2\pi, 2\pi \rangle$ , v bodě  $\frac{1}{3}\pi$  a její detail v blízkosti bodu dotyku  $f(a) = f(\frac{1}{3}\pi) = (\frac{1}{3}\pi, \frac{1}{3}\pi - \frac{1}{2}\sqrt{3}) \doteq (1.05, 0.18)$ .



### Cvičení 15.12 a 15.13 na str. 125

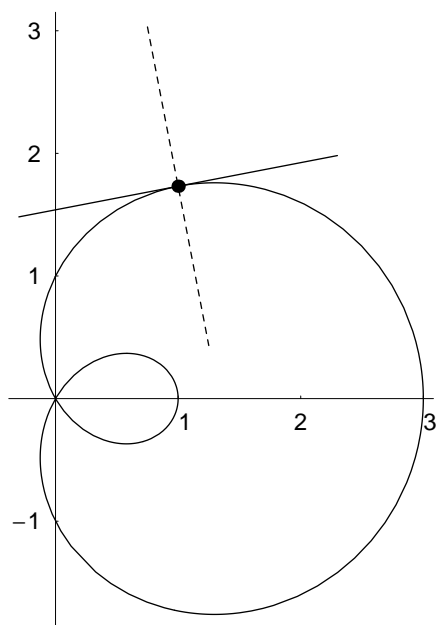
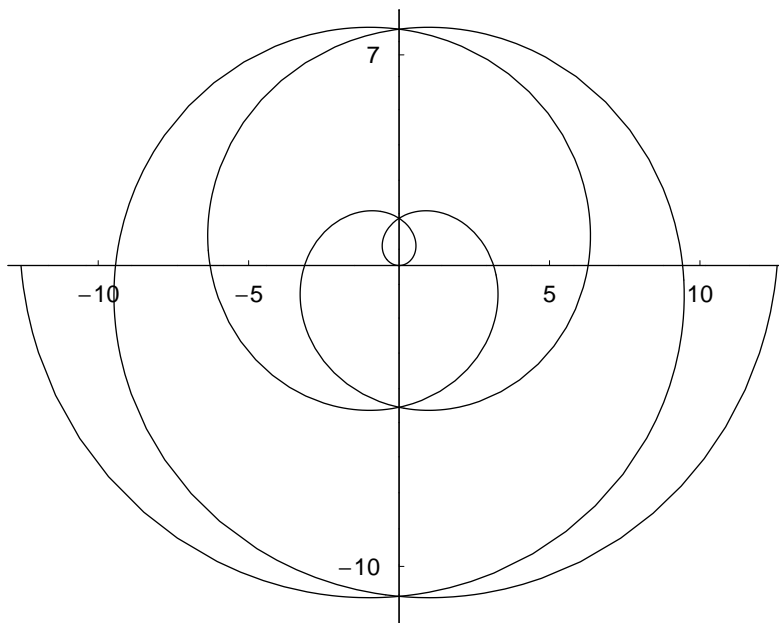
Tečna a normála strofoidy  $f(t) = (\cos 2t, \cos 2t \operatorname{tg} t)$ ,  $t \in \langle -1.3, 1.3 \rangle$ , v bodě  $\frac{1}{4}\pi$  a  
čtyřlístku  $f(t) = (\cos t + \sin 3t, \sin t + \cos 3t)$ ,  $t \in \langle 0, 2\pi \rangle$ , v bodě  $\frac{1}{3}\pi$ .





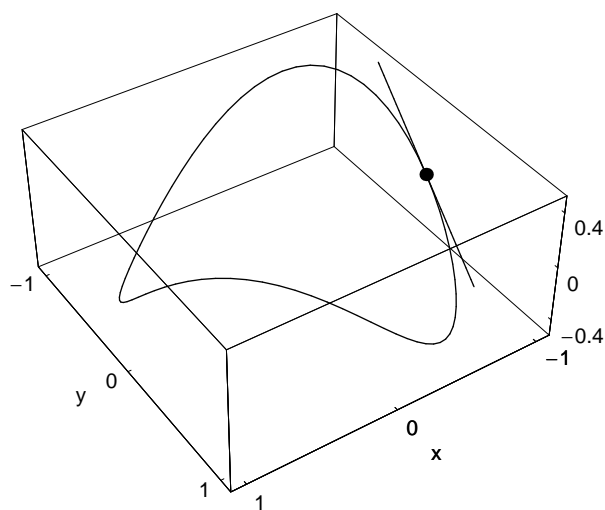
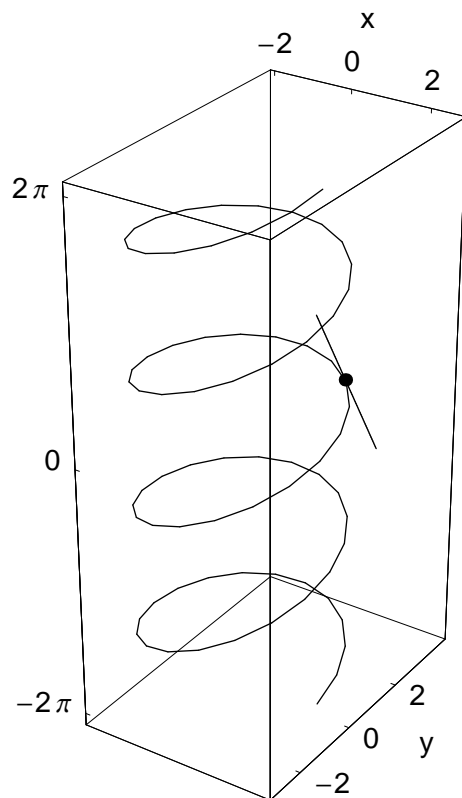
### Cvičení 15.14 a 15.15 na str. 125

Tečna a normála Archimedovy spirály  $f(t) = (t \cos t, t \sin t)$ ,  $t \in \langle -4\pi, 4\pi \rangle$ , v počátku je totožná s osou x, resp. y;  
 tečna a normála Pascalovy závitnice  $f(t) = (2 \cos t + 1)(\cos t, \sin t)$ ,  $t \in \langle 0, 2\pi \rangle$ , v bodě  $\frac{1}{3}\pi$ .



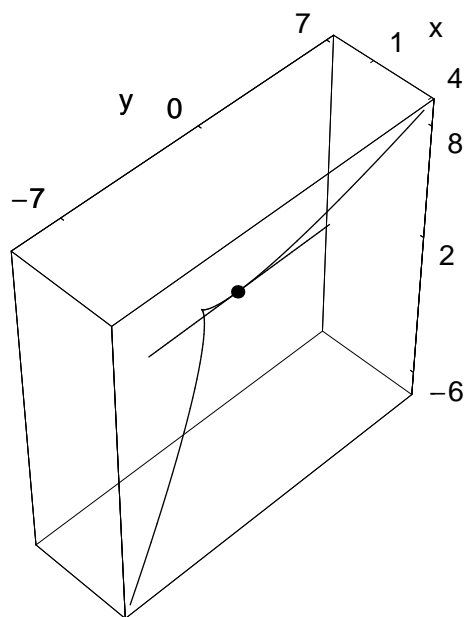
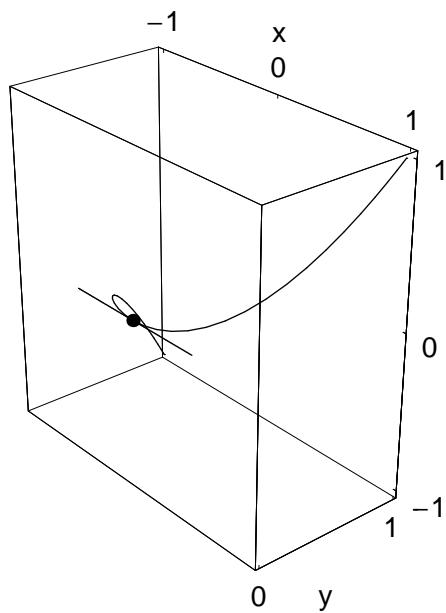
### Cvičení 15.16 a 15.17 na str. 125

Tečna eliptické závitnice  $f(t) = (2 \cos t, 3 \sin t, \frac{1}{2}t)$ ,  $t \in \langle -4\pi, 4\pi \rangle$ , v bodě  $\frac{1}{3}\pi$  a  
křivky  $f(t) = (\cos t, \sin t, \sin t \cos t)$ ,  $t \in \langle 0, 2\pi \rangle$ , v bodě  $\pi$ .



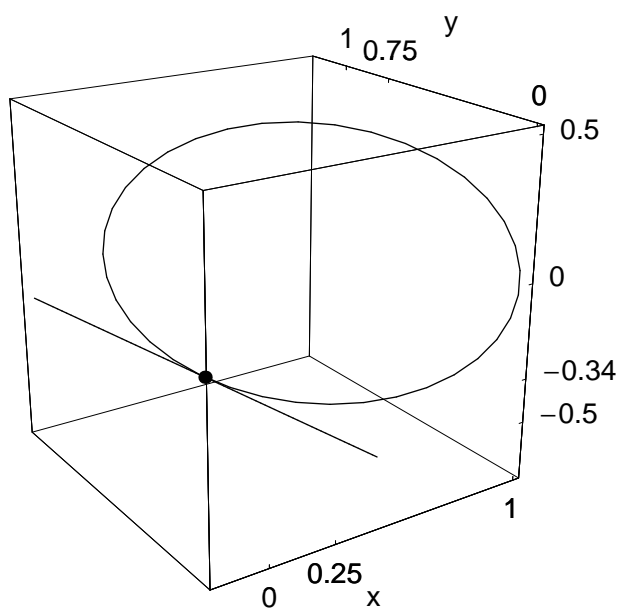
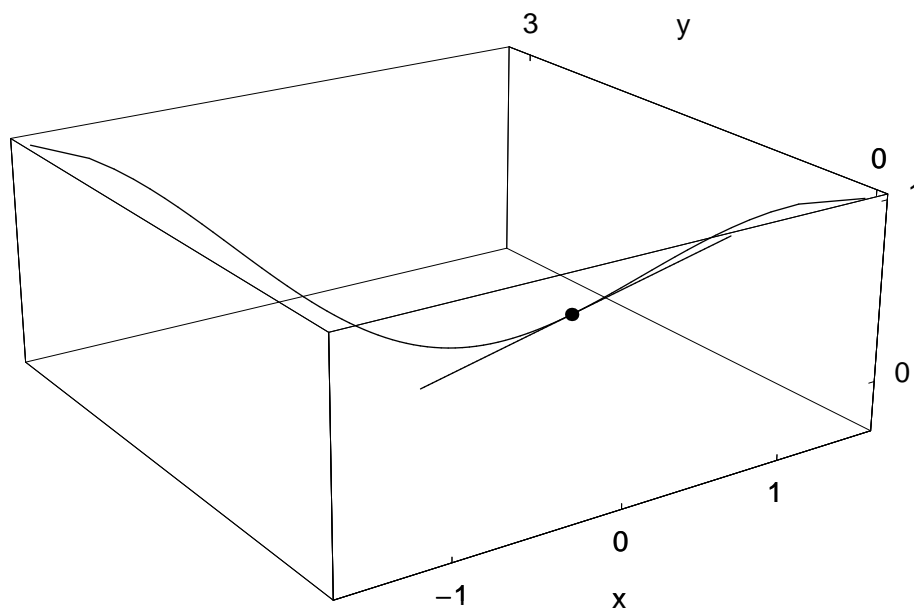
### Cvičení 15.18 a 15.19 na str. 126

Tečna křivky  $f(t) = (\sin t, \sin^2 t, \sin^3 t)$ ,  $t \in \langle 0, 2\pi \rangle$ , v bodě  $\pi$  a  
křivky  $f(t) = (t^2, t^3 - 1, t^3 + 1)$ ,  $t \in \langle 0, 2\pi \rangle$ , v bodě  $\pi$ .



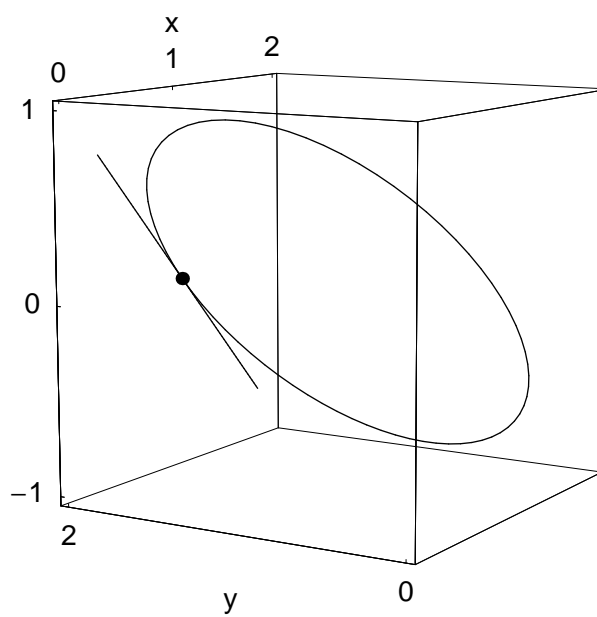
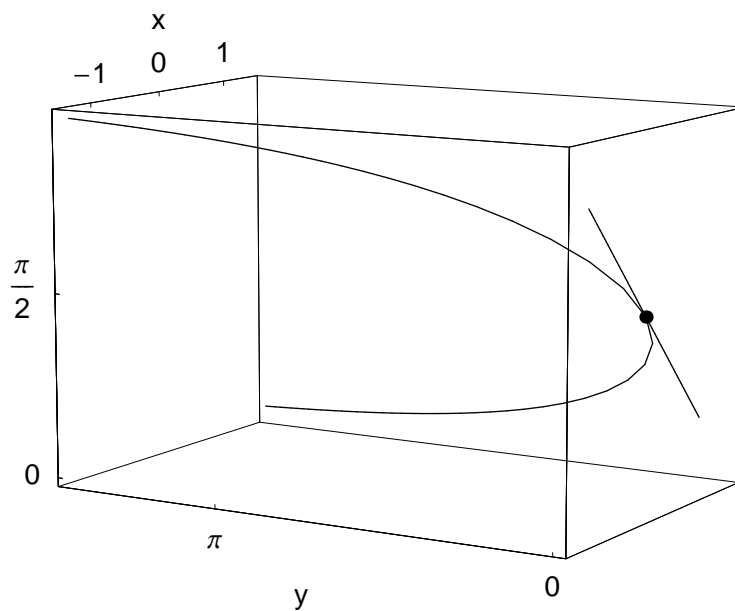
### Cvičení 15.20 a 15.21 na str. 126

Tečna křivky  $f(t) = (\arcsin t, \arccos t, t^2)$ ,  $t \in \langle -1, 1 \rangle$ , v bodě  $\frac{1}{2}$  a  
křivky  $f(t) = (\sin^2 t, \cos^2 t, \sin t \cos t)$ ,  $t \in \langle 0, 2\pi \rangle$ , v bodě  $\frac{5}{6}\pi$ .



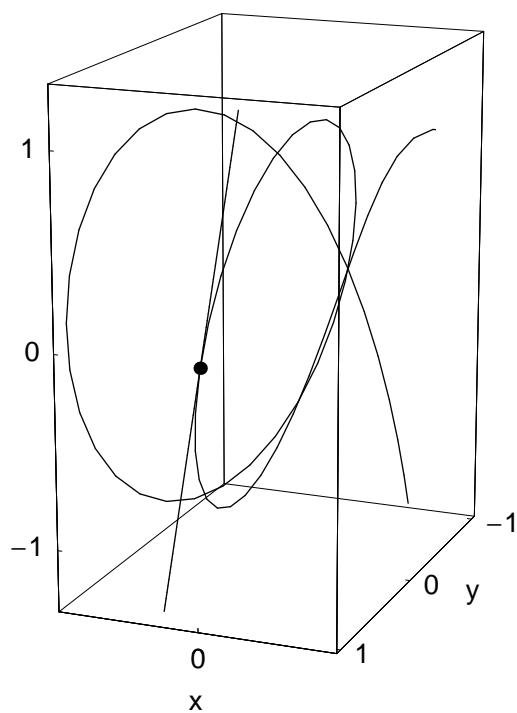
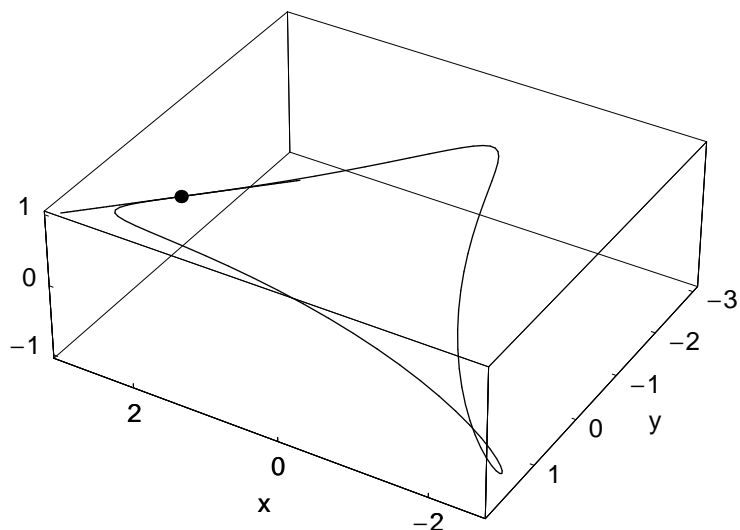
### Cvičení 15.22 a 15.23 na str. 126

Tečna křivky  $f(t) = (\operatorname{arctg} t, \lg(1+t^2), \operatorname{arccot} t)$ ,  $t \in \langle -10, 10 \rangle$ , v bodě 0 a  
křivky  $f(t) = (t - \sin t, 1 - \cos t, \sin t)$ ,  $t \in \langle 0, 2\pi \rangle$ , v bodě  $\pi$ .



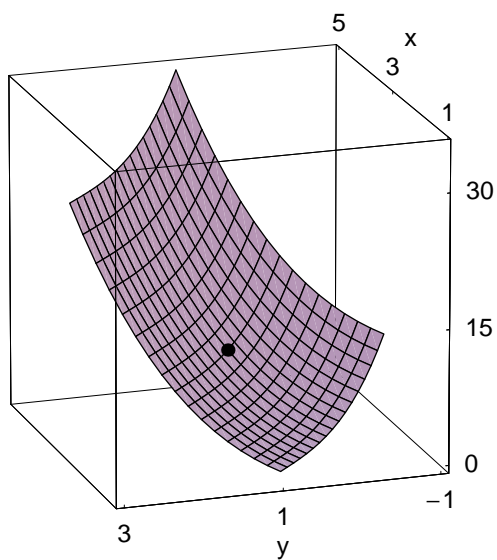
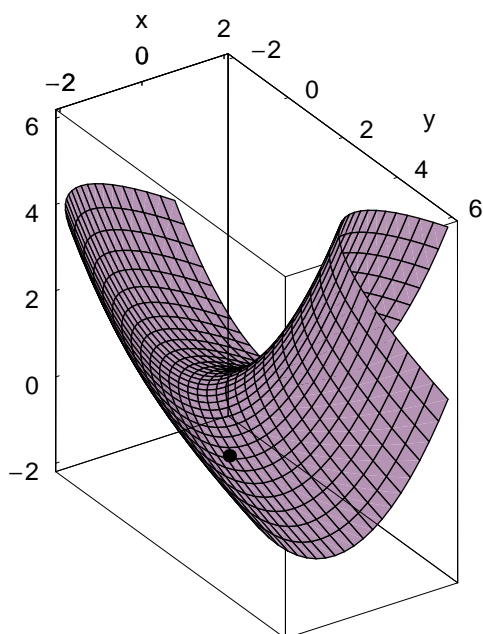
### Cvičení 15.24 a 15.25 na str. 126

Tečna křivky  $f(t) = (2 \sin t + \sin 2t, 2 \cos t - \cos 2t, \sin t)$ ,  $t \in \langle 0, 2\pi \rangle$ , v bodě  $\frac{1}{2}\pi$  a křivky  $f(t) = (\lg t \cos \pi t, \cos 2\pi t, \sin 2\pi t)$ ,  $t \in \langle 0, 2\pi \rangle$ , v bodě 1.



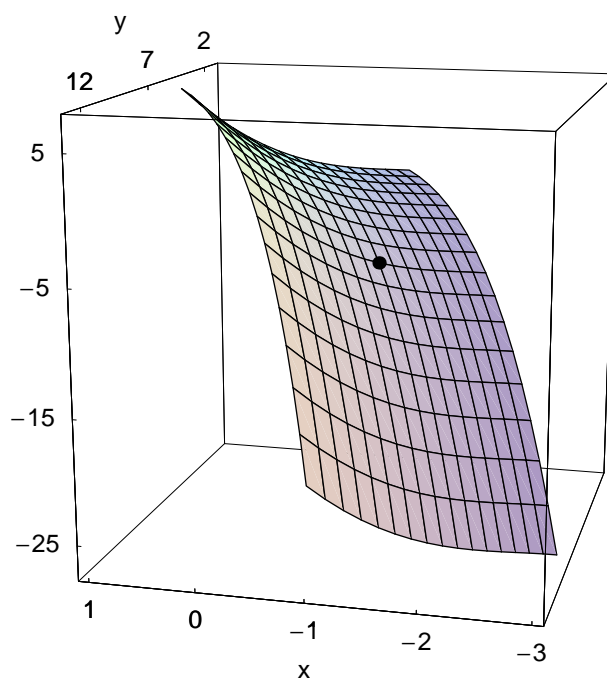
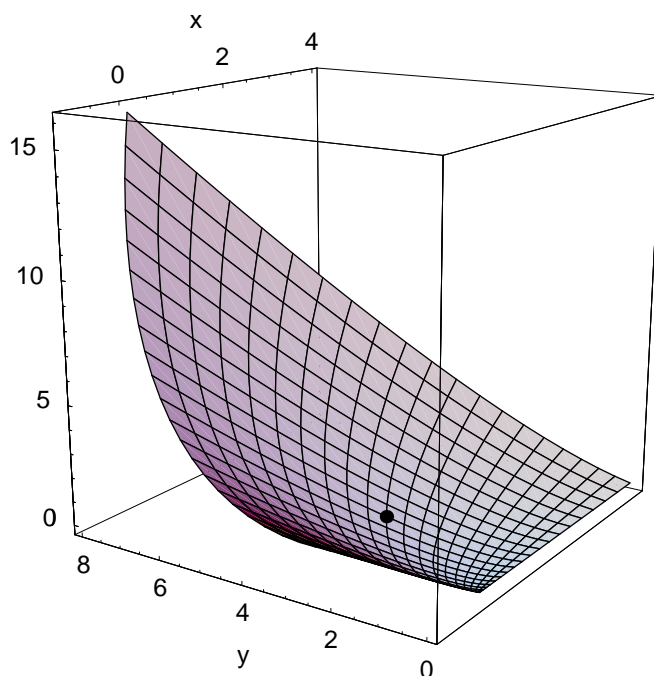
### Cvičení 15.31 a 15.32 na str. 126

Plocha  $f(u, v) = (u, u + v^2, u^2 - v)$ ,  $(u, v) \in \langle -2, 2 \rangle^2$ ,  $a = (0, 1)$  a  
 $f(u, v) = (u + v, u - v, u^3 + v^3)$ ,  $(u, v) \in \langle 1, 3 \rangle \times \langle 0, 2 \rangle$ ,  $a = (2, 1)$ .



**Cvičení 15.33 a 15.34 na str. 126**

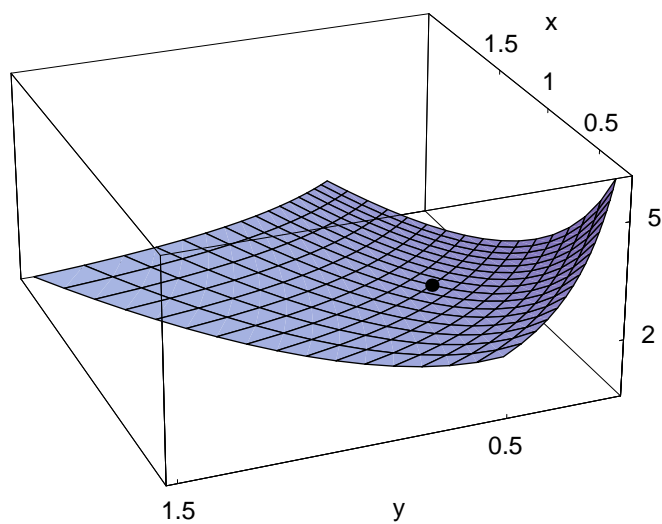
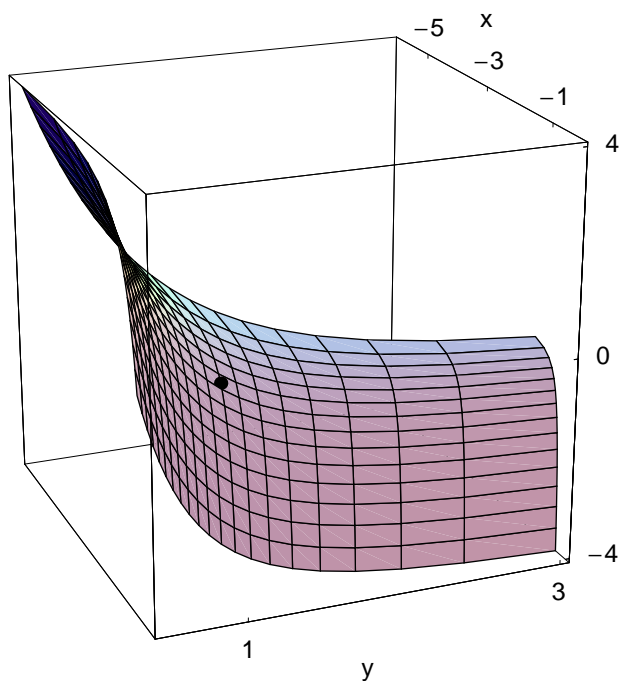
Plocha  $f(u, v) = (u^2 - uv, uv + v^2, u^2v + uv^2)$ ,  $(u, v) \in \langle 0, 2 \rangle^2$ ,  $a = (1, 1)$  a  
 $f(u, v) = (u + v, u^2 + v^2, u^3 + v^3)$ ,  $(u, v) \in \langle 0, 2 \rangle \times \langle -3, -1 \rangle$ ,  $(a = 1, -2)$ .





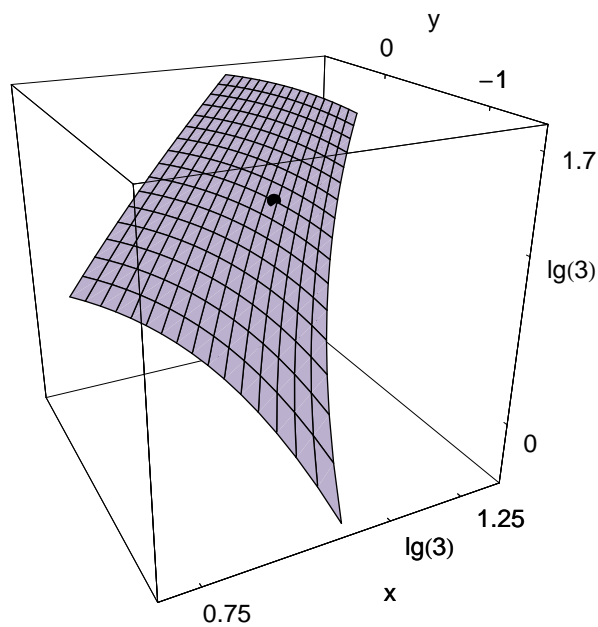
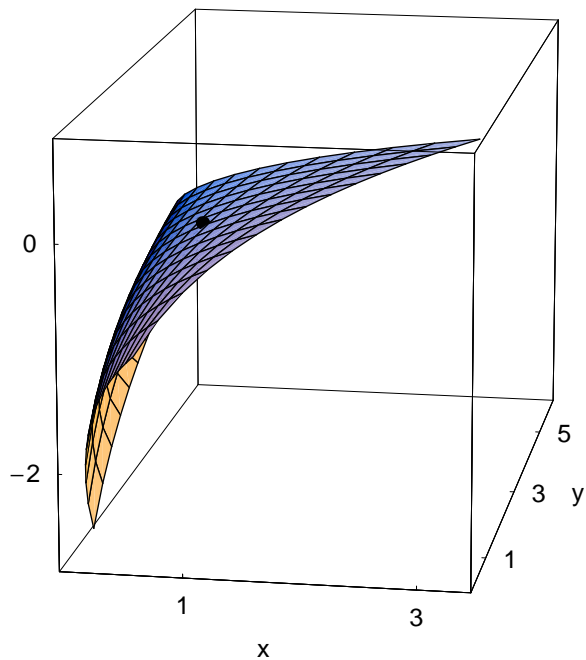
**Cvičení 15.35 a 15.36 na str. 126**

Plocha  $f(u, v) = (u/v, 1/u, u^2 - v^2)$ ,  $(u, v) \in \langle \frac{1}{3}, 2 \rangle \times \langle -2, -\frac{1}{3} \rangle$ ,  $a = (1, -1)$  a  
 $f(u, v) = (u^2, u/v, v/u)$ ,  $(u, v) \in \langle -\frac{3}{2}, -\frac{1}{2} \rangle \times \langle -3, -1 \rangle$ ,  $a = (-1, -2)$ .



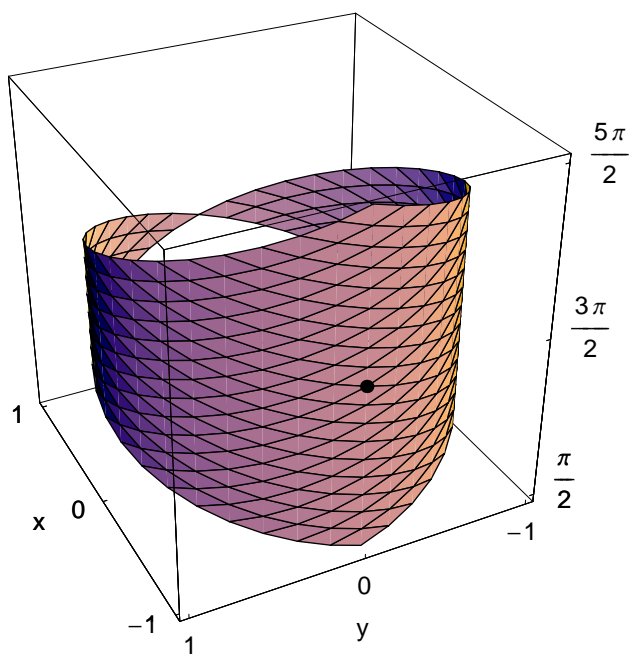
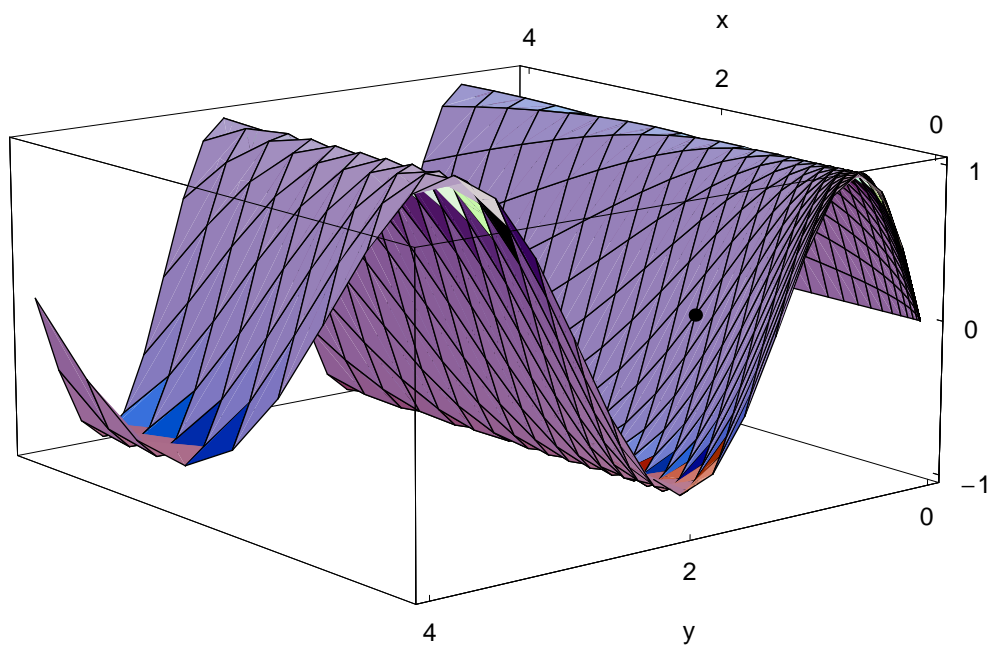
**Cvičení 15.37 a 15.38 na str. 126**

Plocha  $f(u, v) = (uv^2, u/v, \lg(uv))$ ,  $(u, v) \in \langle \frac{1}{4}, \frac{3}{2} \rangle^2$ ,  $a = (1, 1)$  a  
 $f(u, v) = (\lg(u + v), \lg(u - v), \lg(u^2 - v^2))$ ,  $(u, v) \in \langle \frac{3}{2}, \frac{5}{2} \rangle \times \langle \frac{1}{2}, \frac{5}{4} \rangle$ ,  $a = (2, 1)$ .



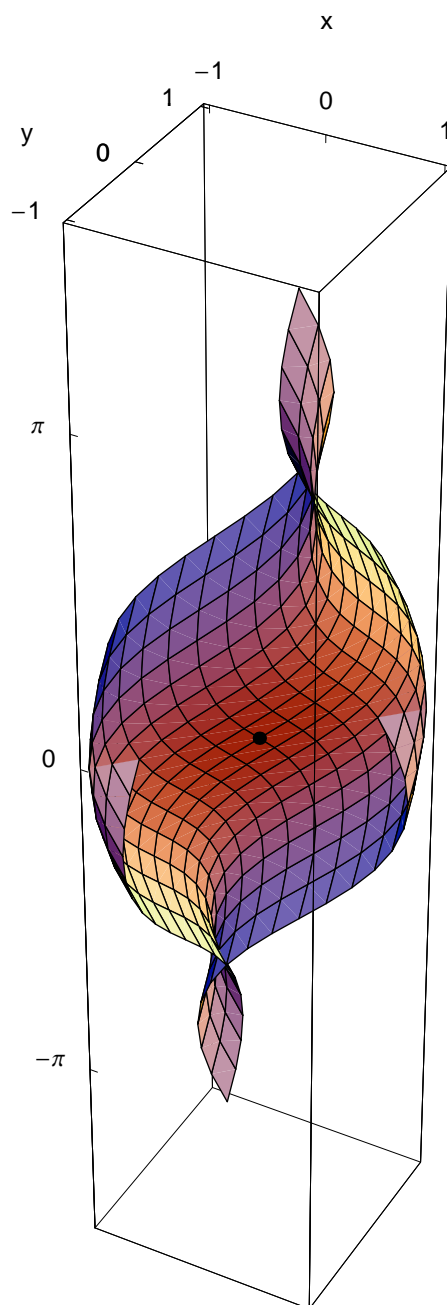
**Cvičení 15.39 a 15.41 na str. 126**

Plocha  $f(u, v) = (u^2, uv, \sin(\pi uv))$ ,  $(u, v) \in \langle 0, 2 \rangle^2$ ,  $a = (1, 1)$  a  
 $f(u, v) = (\sin(u - v), \cos(u - v), u + v)$ ,  $(u, v) \in \langle 0, \pi \rangle \times \langle \frac{1}{2}\pi, \frac{3}{2}\pi \rangle$ ,  $a = (\frac{1}{2}\pi, \pi)$ .



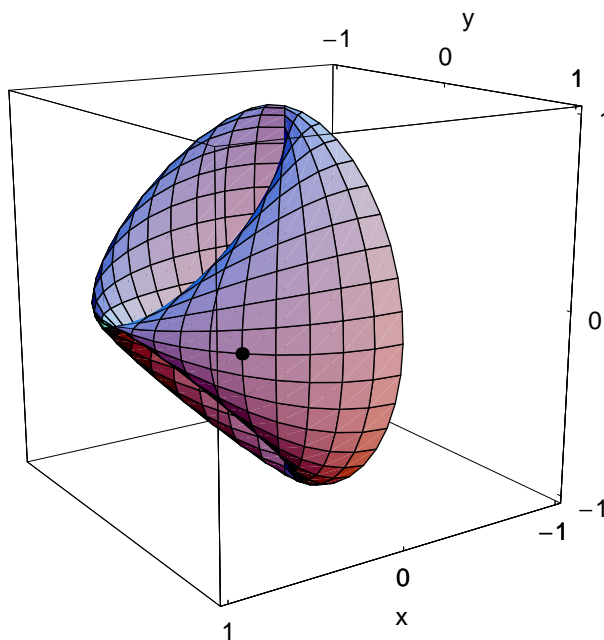
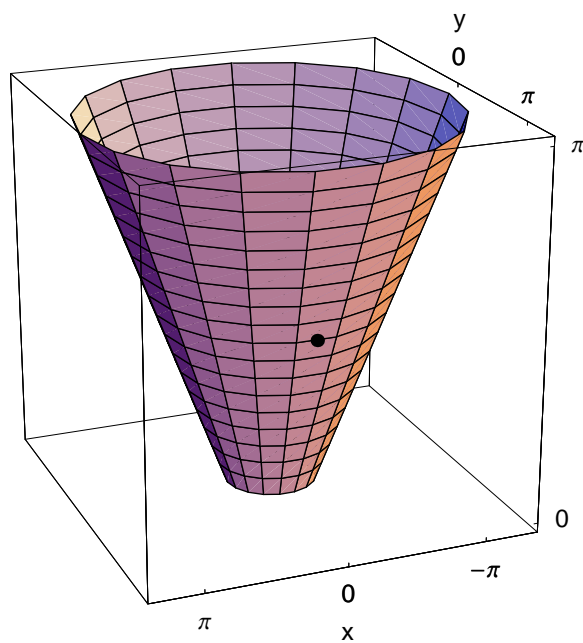
### Cvičení 15.40 na str. 126

Plocha  $f(u, v) = (\sin u, \sin v, u - v)$ ,  $(u, v) \in \langle -\frac{3}{4}\pi, \frac{3}{4}\pi \rangle^2$ ,  $a = (0, 0)$ .



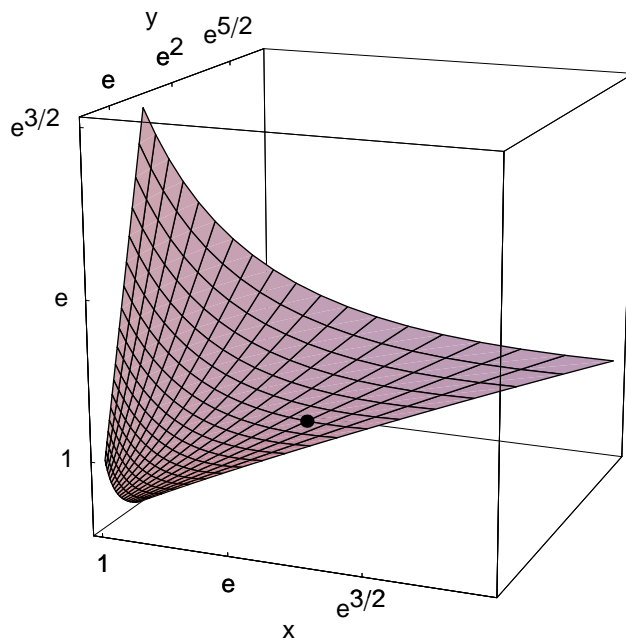
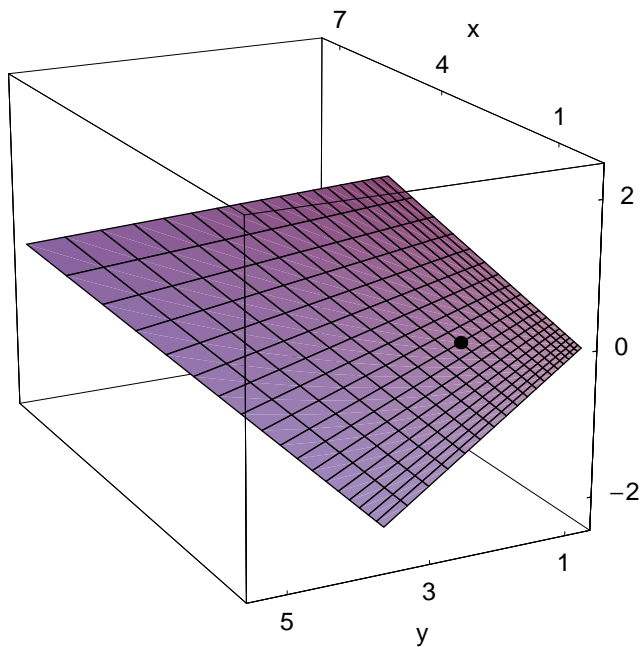
### Cvičení 15.42 a 15.43 na str. 127

Plocha  $f(u, v) = ((1 + |u|) \cos v, (1 + |u|) \sin v, |u|)$ ,  $(u, v) \in \langle 0, \pi \rangle \times \langle 0, 2\pi \rangle$ ,  $a = (\frac{1}{2}\pi, \frac{1}{2}\pi)$  a  
 $f(u, v) = (\sin u \sin v, \sin u \cos v, \cos u \cos v)$ ,  $(u, v) \in \langle 0, \pi \rangle^2$ ,  $a = (\frac{1}{2}\pi, \frac{1}{4}\pi)$ .



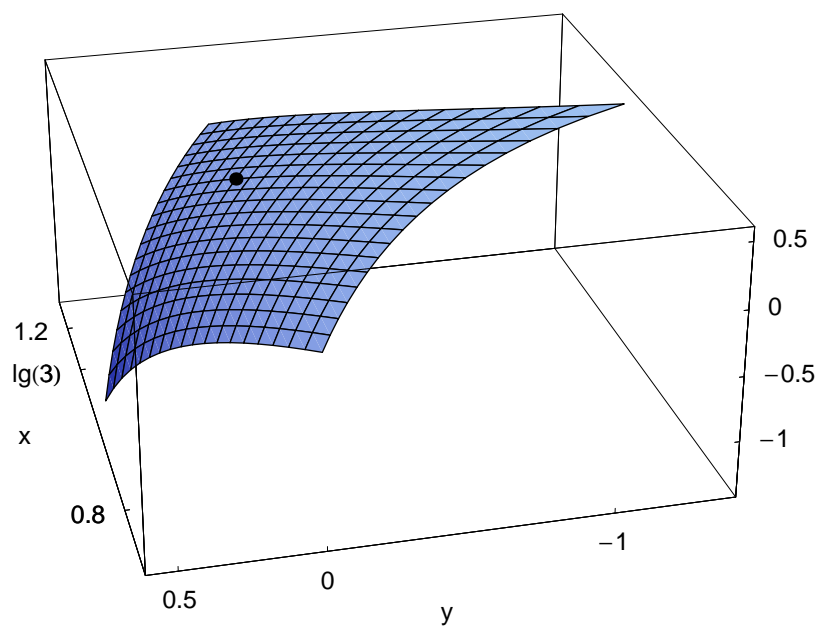
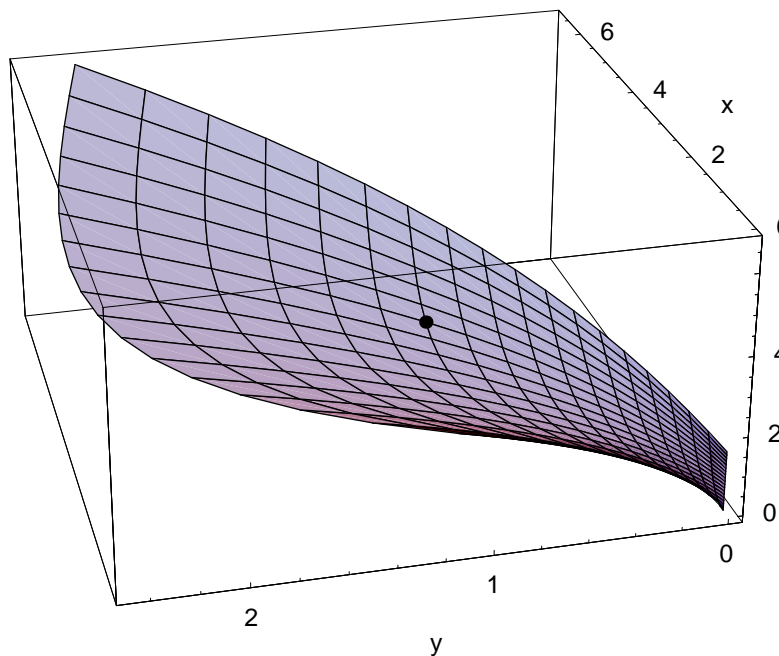
**Cvičení 15.44 a 15.45 na str. 127**

Plocha  $f(u, v) = (e^{u+v}, e^u + e^v, e^u - e^v)$ ,  $(u, v) \in \langle -1, 1 \rangle^2$ ,  $a = (0, 0)$  a  
 $f(u, v) = (e^{uv}, e^{u+v}, e^{u-v})$ ,  $(u, v) \in \langle 0, \frac{3}{2} \rangle \times \langle 0, \frac{6}{5} \rangle$ ,  $a = (1, 1)$ .



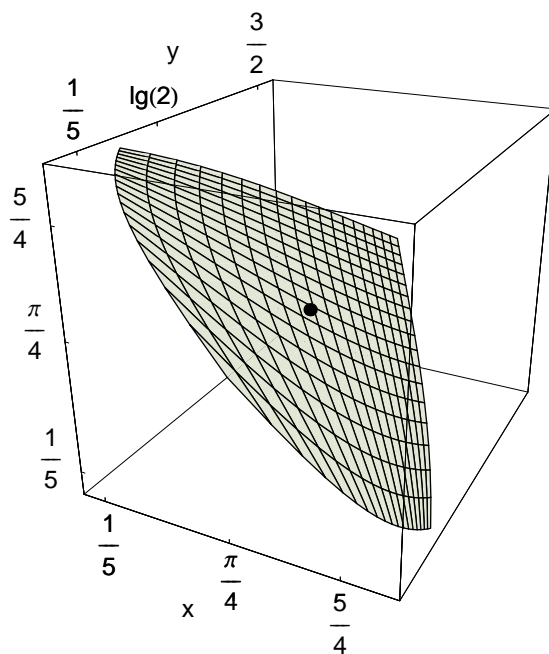
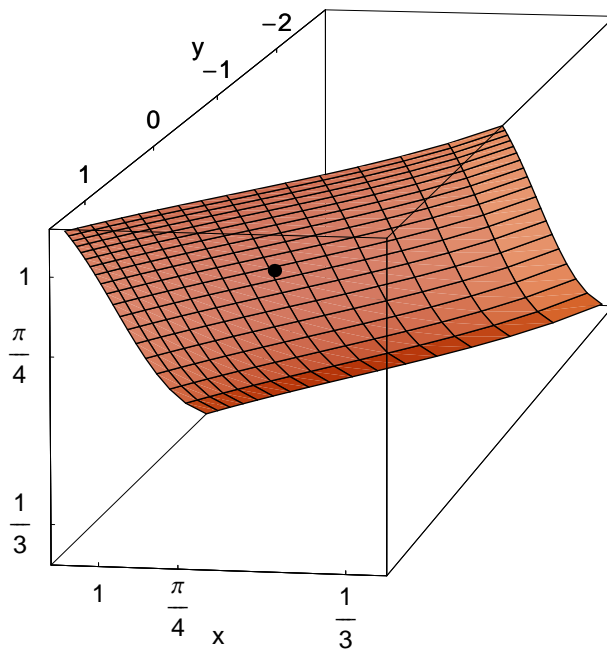
### Cvičení 15.46 a 15.47 na str. 127

Plocha  $f(u, v) = (ue^v, uve^{u-v}, ve^u)$ ,  $(u, v) \in \langle 0, \frac{3}{2} \rangle^2$ ,  $a = (1, 1)$  a  
 $f(u, v) = (\lg(1+u+v), \lg(1-u+v), \lg(1+u-v))$ ,  $(u, v) \in \langle \frac{1}{2}, \frac{5}{4} \rangle^2$ ,  $a = (1, 1)$ .



### Cvičení 15.48 a 15.49 na str. 127

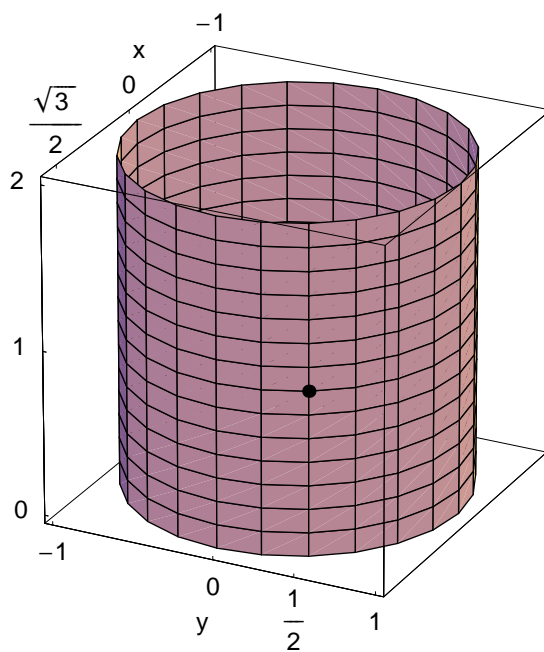
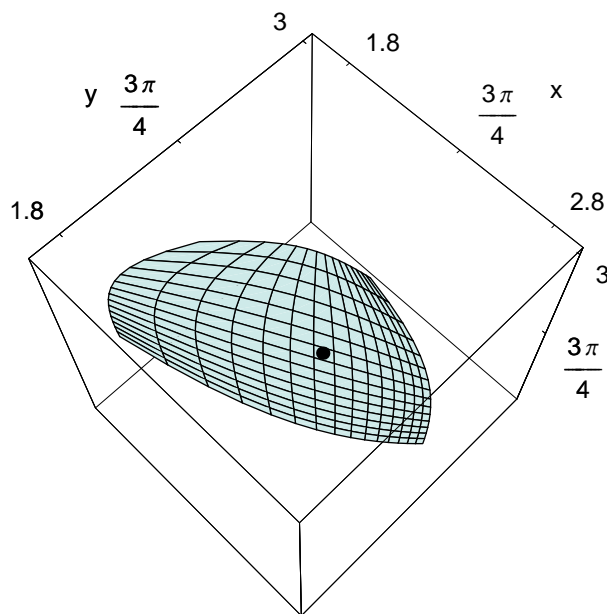
Plocha  $f(u, v) = (\arctg u, \lg(uv), \arctg v)$ ,  $(u, v) \in \langle \frac{1}{4}, 2 \rangle^2$ ,  $a = (1, 1)$  a  
 $f(u, v) = (\arctg(u/v), \lg(1 + uv), \arctg(v/u))$ ,  $(u, v) \in \langle \frac{1}{4}, 2 \rangle^2$ ,  $a = (1, 1)$ .





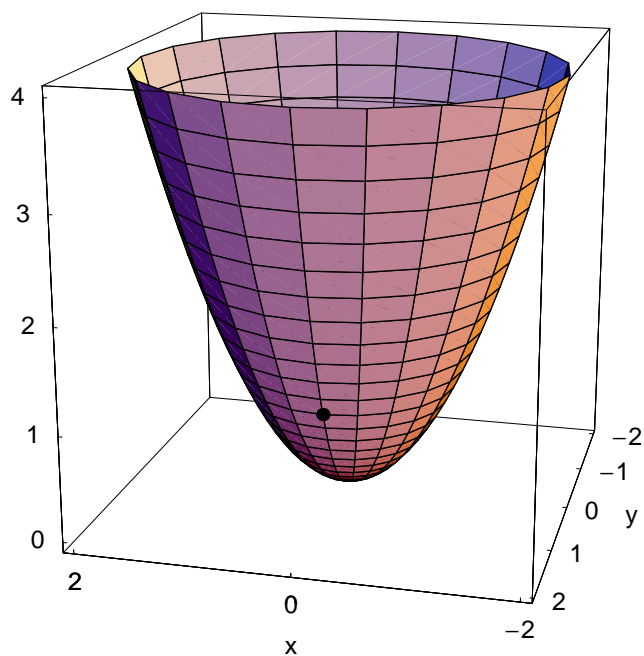
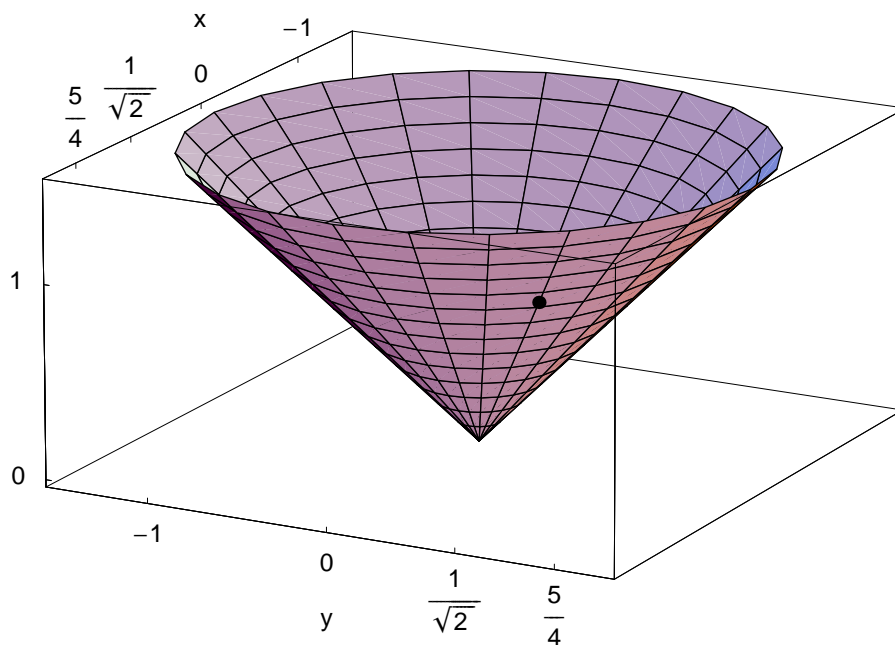
### Cvičení 15.50 a 15.51 na str. 127

Plocha  $f(u, v) = (\operatorname{arccotg}(uv), \operatorname{arccotg}(u/v), \operatorname{arccotg}(v/u))$ ,  $(u, v) \in \langle \frac{1}{4}, 2 \rangle \times \langle -2, -\frac{1}{4} \rangle$ ,  $a = (1, -1)$  a  
 válec  $f(u, v) = (\cos v, \sin v, u)$ ,  $(u, v) \in \langle 0, 2 \rangle \times \langle 0, 2\pi \rangle$ ,  $a = (1, \frac{1}{6}\pi)$ .



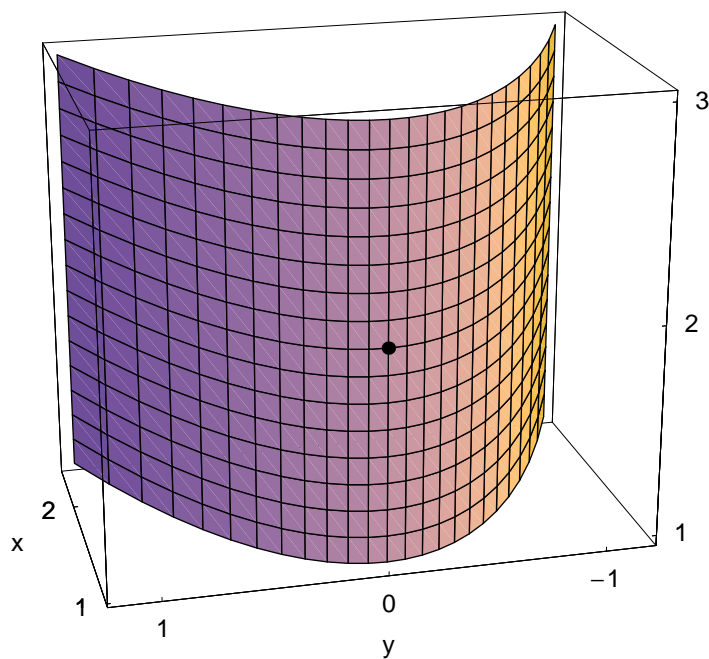
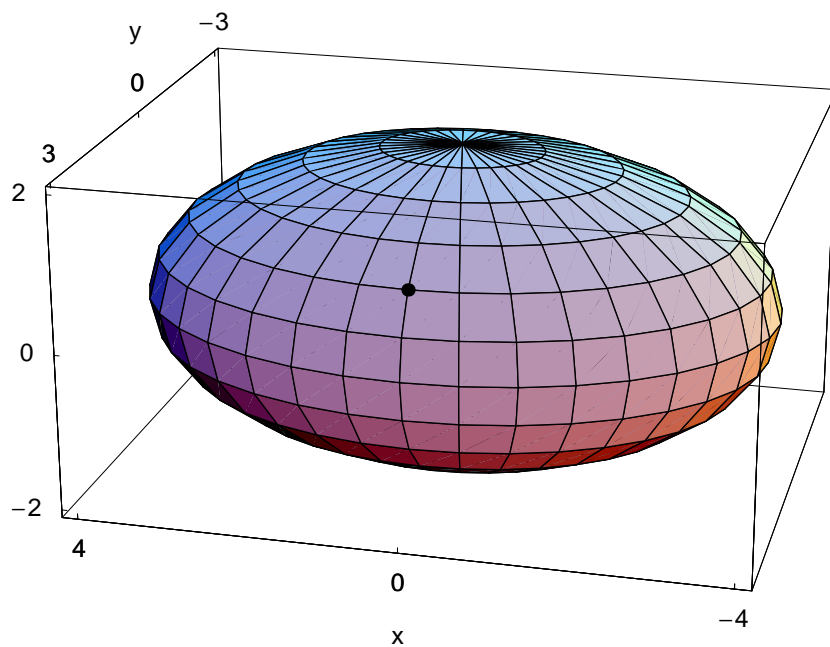
**Cvičení 15.52 a 15.53 na str. 127**

Kužel  $f(u, v) = (u \cos v, u \sin v, u)$ ,  $(u, v) \in \langle 0, \frac{3}{2} \rangle \times \langle 0, 2\pi \rangle$ ,  $a = (1, \frac{1}{4}\pi)$  a  
 paraboloid  $f(u, v) = (v \sin u, v \cos u, v^2)$ ,  $(u, v) \in \langle 0, 2\pi \rangle \times \langle 0, 2 \rangle$ ,  $a = (0, 1)$ .



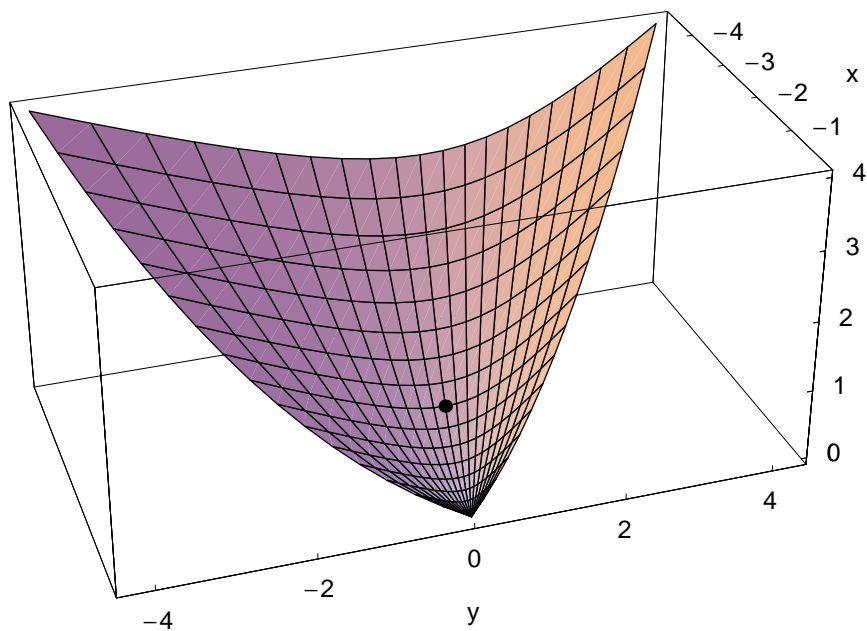
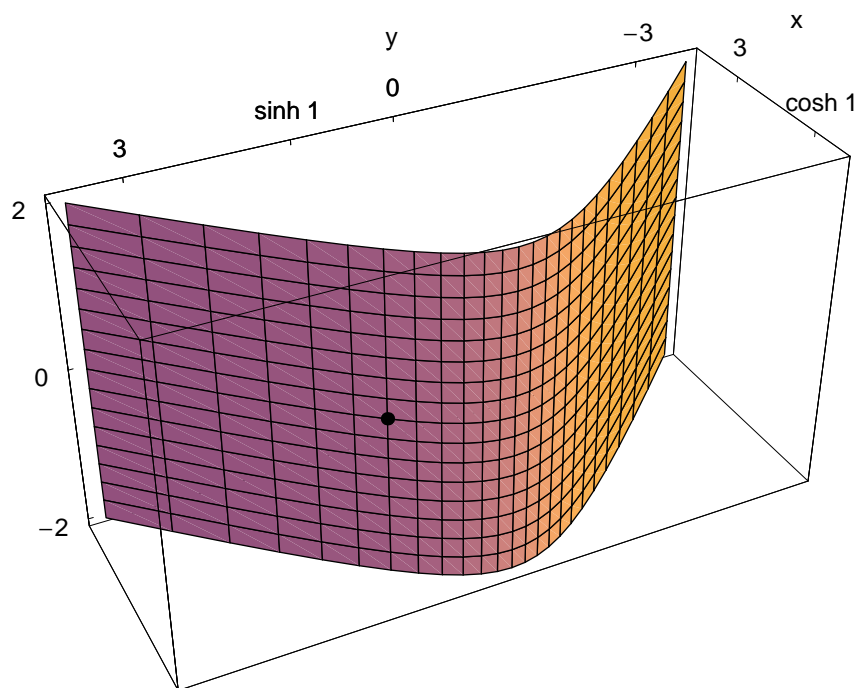
### Cvičení 15.54 a 15.55 na str. 127

Elipsoid  $f(u, v) = (4 \cos u \cos v, 3 \sin u \cos v, 2 \sin v)$ ,  $(u, v) \in \langle 0, \pi \rangle \times \langle 0, 2\pi \rangle$ ,  $a = (\frac{1}{2}\pi, \frac{1}{6}\pi)$  a  
parabolický válec  $f(u, v) = (\cosh^2 v, \sinh v, u)$ ,  $(u, v) \in \langle 1, 3 \rangle \times \langle -1, 1 \rangle$ ,  $a = (2, 0)$ .



### Cvičení 15.56 a 15.57 na str. 127

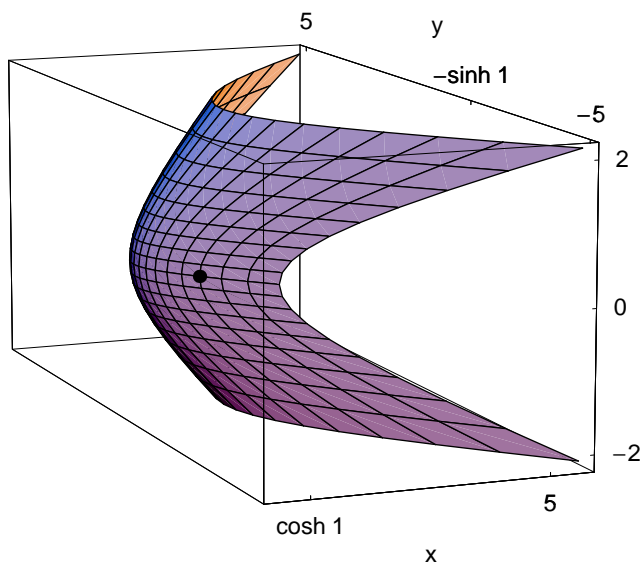
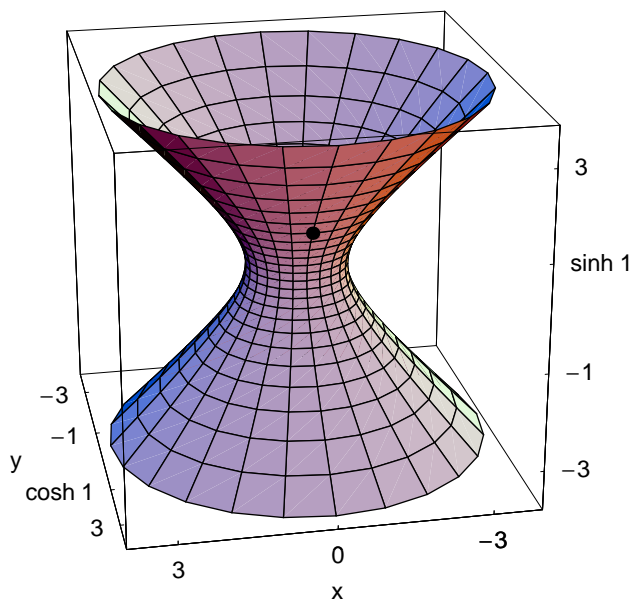
Půl hyperbolického válce  $f(u, v) = (\cosh v, \sinh v, u)$ ,  $(u, v) \in \langle -2, 2 \rangle^2$ ,  $a = (0, 1)$  a část hyperbolického paraboloidu  $f(u, v) = (u \cosh v, u \sinh v, u^2)$ ,  $(u, v) \in \langle -2, 0 \rangle \times \langle -\frac{3}{2}, \frac{3}{2} \rangle$ ,  $a = (-1, 0)$ .



### Cvičení 15.58 a 15.59 na str. 127

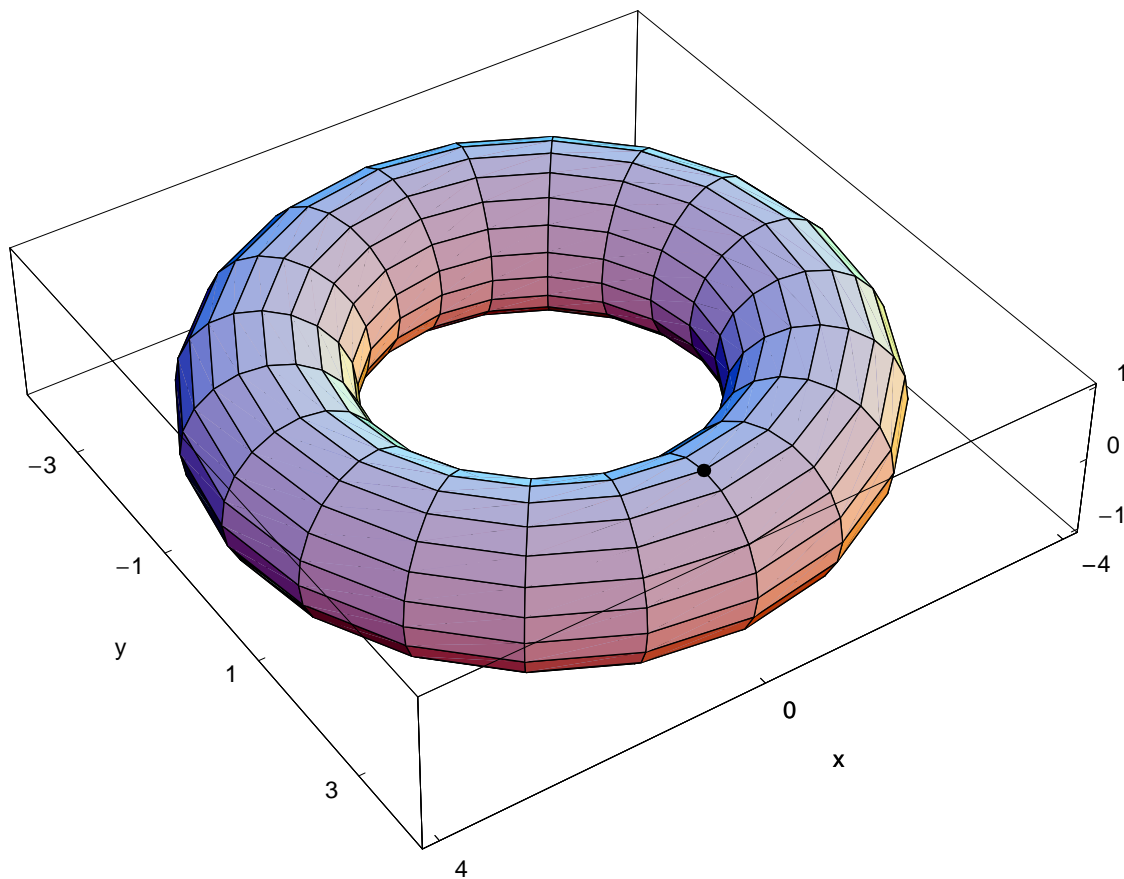
Jednodílný hyperboloid  $f(u, v) = (\cos u \cosh v, \sin u \cosh v, \sinh v)$ ,  $(u, v) \in \langle 0, 2\pi \rangle \times \langle -2, 2 \rangle$ ,  
 $a = (\frac{1}{2}\pi, 1)$  a půl dvojdílného hyperboloidu

$f(u, v) = (\cosh u \cosh v, \sinh u \cosh v, \sinh v)$ ,  $(u, v) \in \langle -\frac{3}{2}, \frac{3}{2} \rangle^2$ ,  $a = (-1, 0)$ .



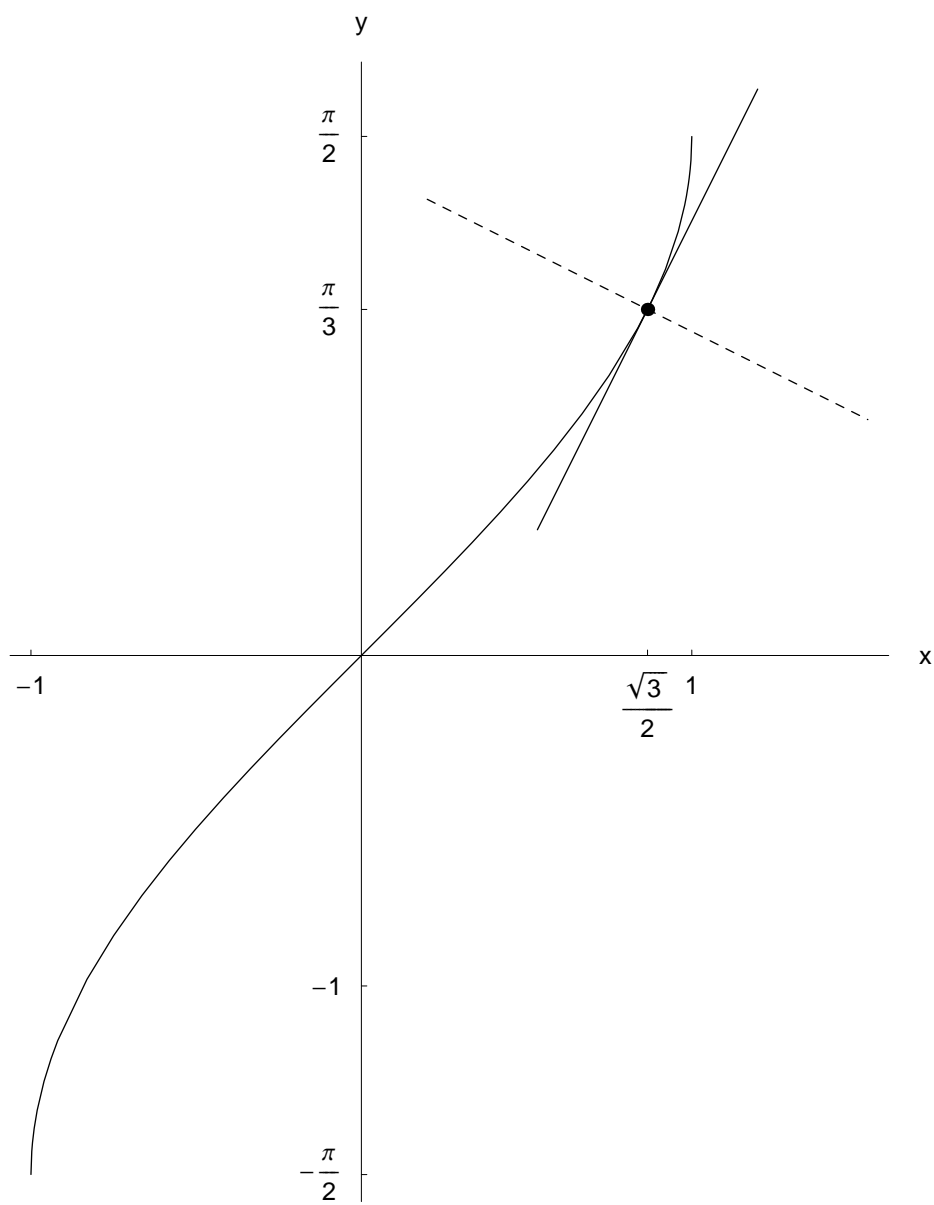
### Cvičení 15.60 na str. 127

Anuloid  $f(u, v) = ((3 + \cos u) \cos v, (3 + \cos u) \sin v, \sin u)$ ,  $(u, v) \in \langle 0, 2\pi \rangle^2$ ,  $a = (\frac{1}{2}\pi, \frac{1}{2}\pi)$ .



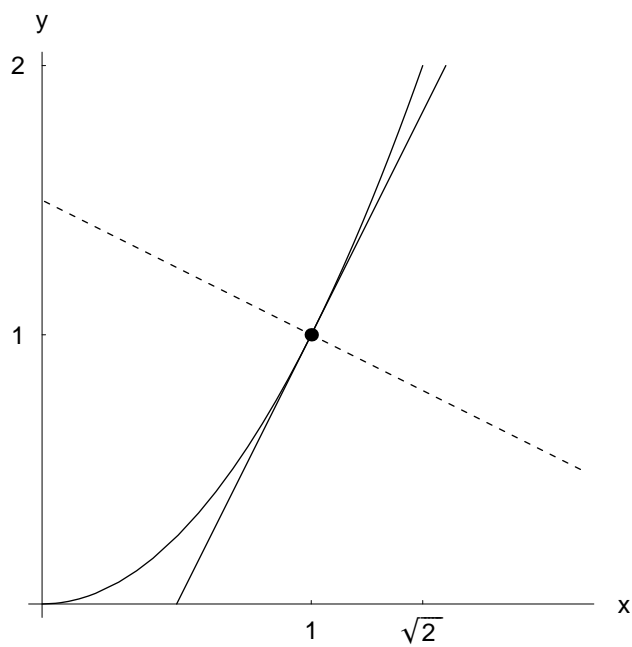
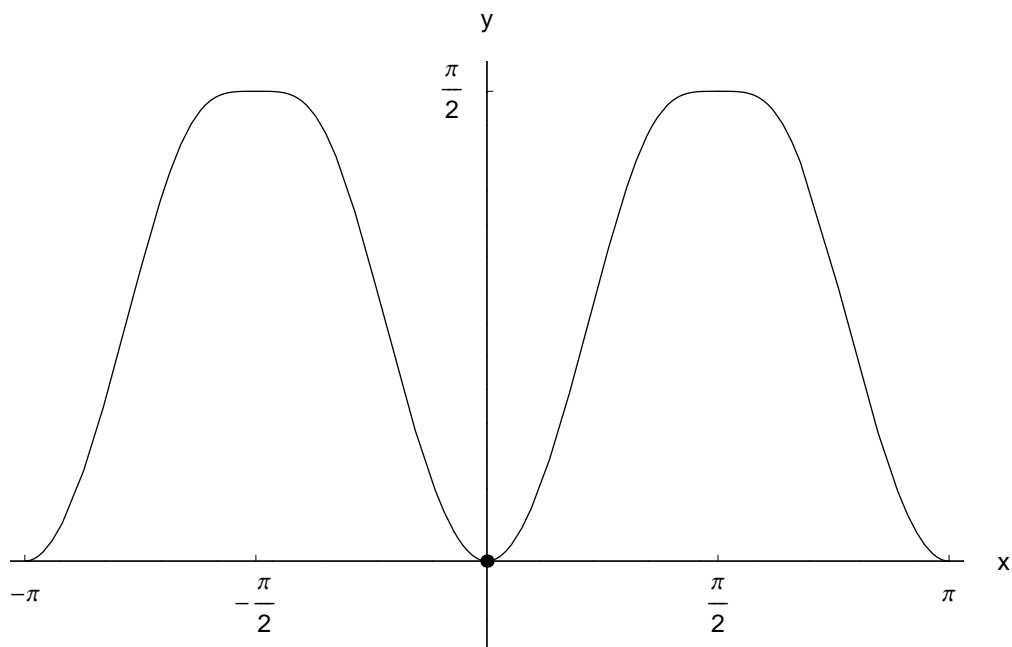
### Cvičení 15.76 na str. 128

Tečna a normála křivky  $y = \arcsin x$ ,  $x \in \langle -1, 1 \rangle$ , v bodě  $\frac{1}{2}\sqrt{3}$ .



### Cvičení 15.77 a 15.78 na str. 128 a 129

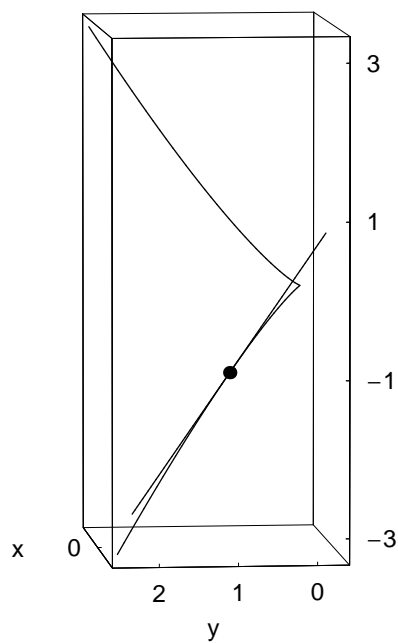
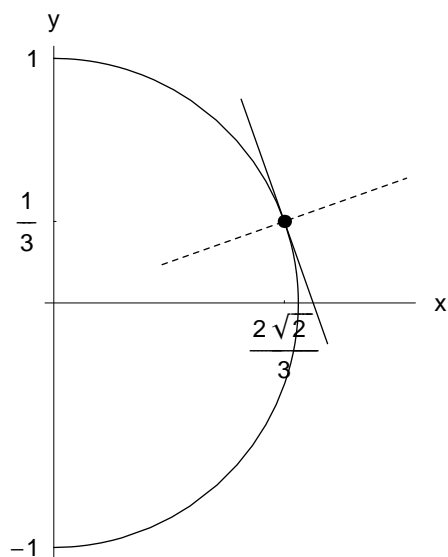
Tečna a normála křivky  $y = \arccos(1 - \sin^4 x)^2$ ,  $x \in \langle -\pi, \pi \rangle$ , v bodě 0 je identická s osou x a y;  
tečna a normála křivky  $x = \sqrt{y}$ ,  $y \in \langle 0, 2 \rangle$ , v bodě 1.





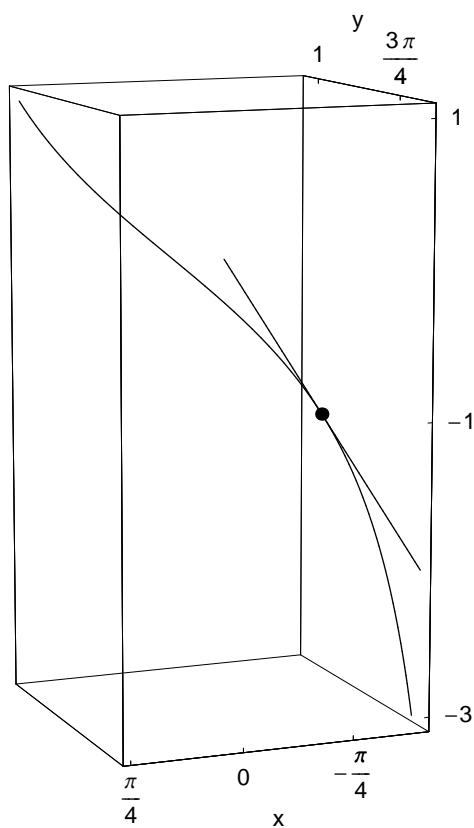
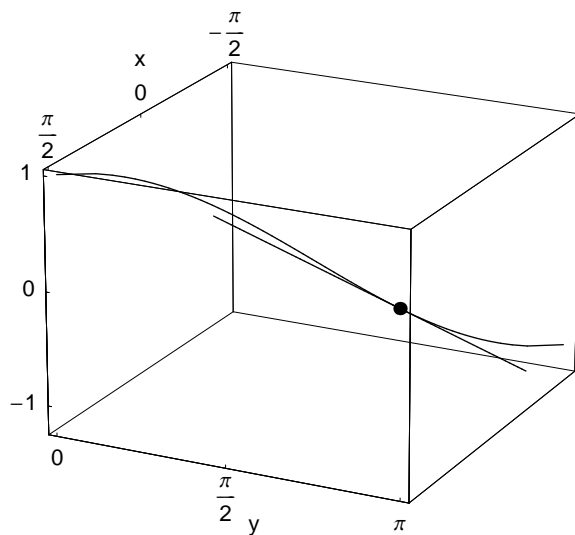
**Cvičení 15.79 a 15.80 na str. 129**

Tečna a normála křivky  $x = \sqrt{1 - y^2}$ ,  $y \in \langle -1, 1 \rangle$ , v bodě  $\frac{1}{3}$  a  
 tečna křivky  $(y, z) = (x^{4/3}, x^{5/3})$ ,  $x \in \langle -2, 2 \rangle$ , v bodě  $-1$ .



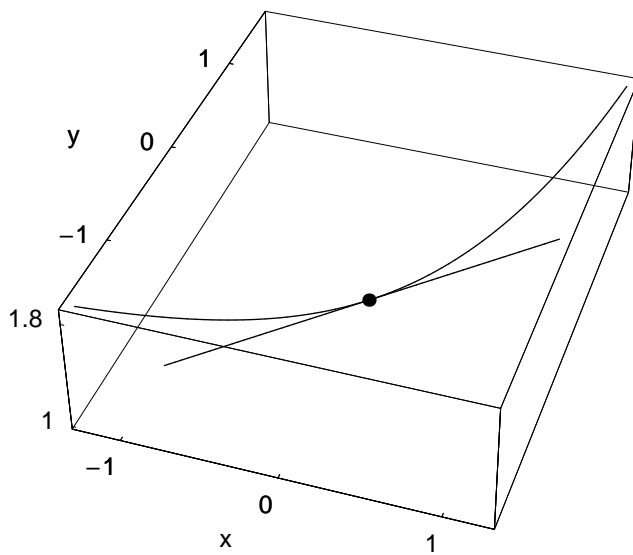
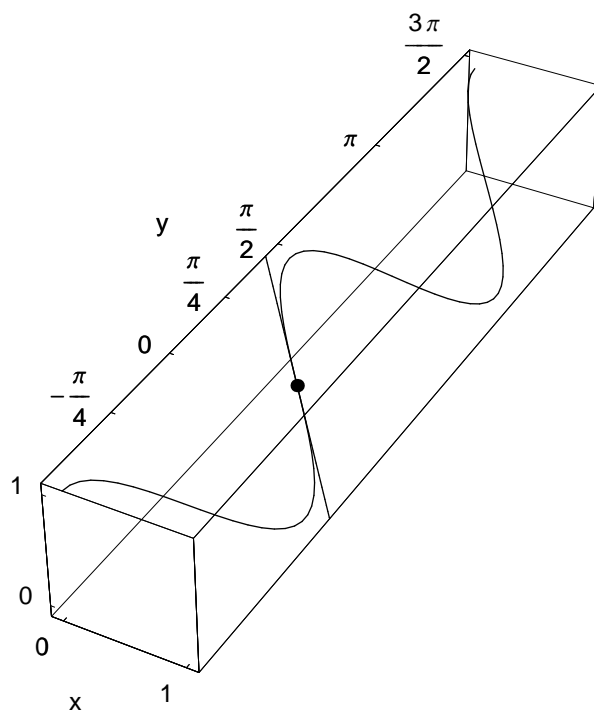
### Cvičení 15.81 a 15.82 na str. 129

Tečna křivky  $(x, y) = (\arcsin z, \arccos z)$ ,  $z \in \langle -1, 1 \rangle$ , v bodě  $-\frac{1}{2}$  a  
 křivky  $(x, y) = (\operatorname{arctg} z, \operatorname{arccotg} z)$ ,  $z \in \langle -3, 1 \rangle$ , v bodě  $-1$ .



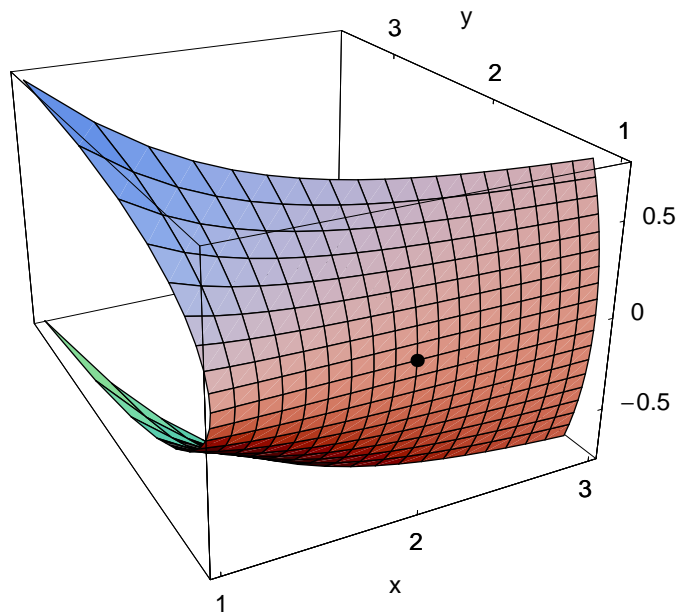
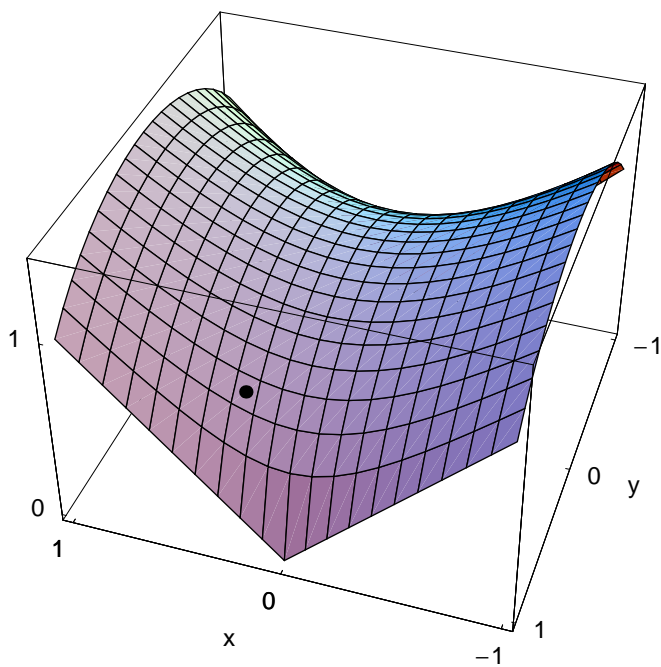
**Cvičení 15.83 a 15.84 na str. 129**

Tečna křivky  $(x, z) = (\cos^2 y, \sin^2 y)$ ,  $y \in \langle -\frac{1}{2}\pi, \frac{3}{2}\pi \rangle$ , v bodě  $\frac{1}{4}\pi$  a  
 křivky  $(y, z) = (\sinh x, \cosh x)$ ,  $x \in \langle -\frac{5}{4}, \frac{5}{4} \rangle$ , v bodě 0.



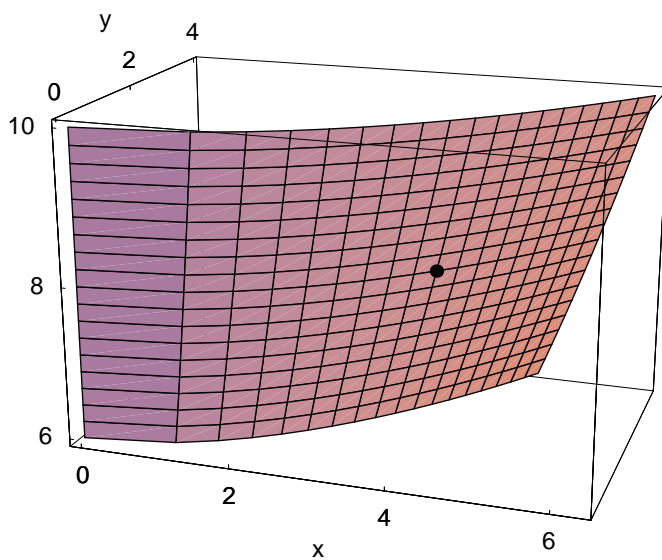
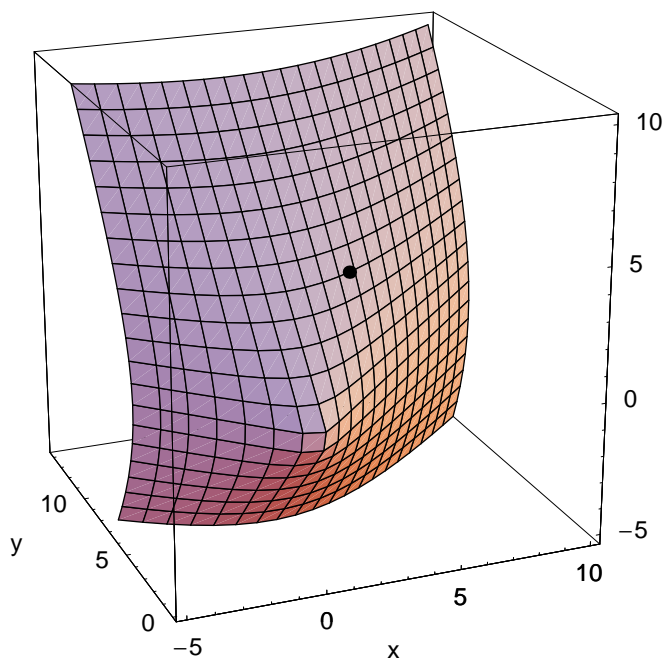
### Cvičení 15.88 a 15.89 na str. 129

Část 1-dílného hyperboloidu  $z = \sqrt{1 + x^2 - y^2}$ ,  $(x, y) \in \langle -1, 1 \rangle^2$ ,  $a = (\frac{1}{4}, \frac{3}{4})$  a  
plochy  $y = (x^2 + z^2)/(x^2 - z^2)$ ,  $(x, z) \in \langle 1, 3 \rangle \times \langle -\frac{3}{4}, \frac{3}{4} \rangle$ ,  $a = (2, 0)$ .



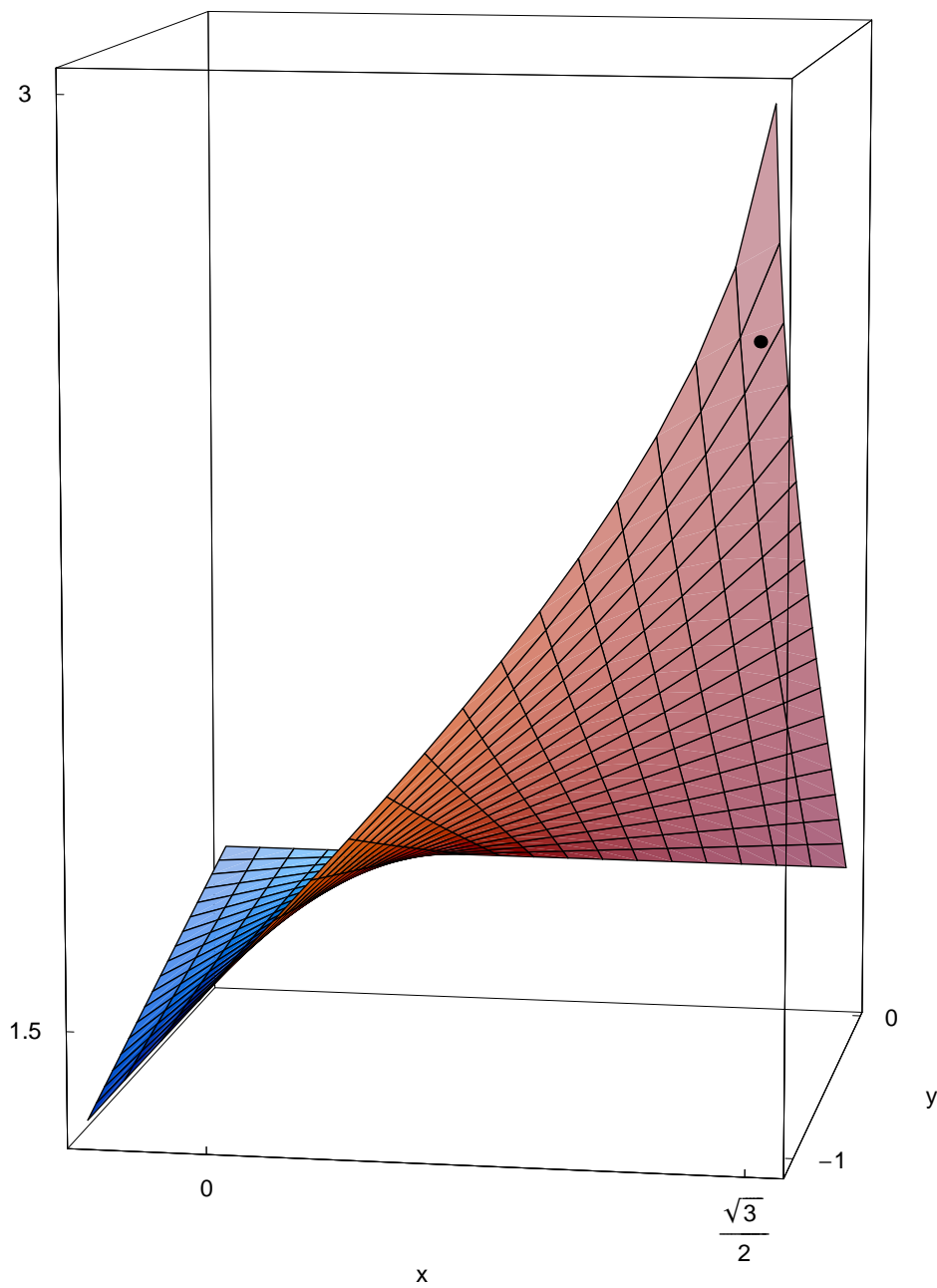
### Cvičení 15.90 a 15.91 na str. 129

Část kuželu  $y = \sqrt{x^2 + z^2}$ ,  $(x, z) \in \langle -5, 10 \rangle^2$ ,  $a = (3, 4)$  a  
plochy  $x = \sqrt{yz}$ ,  $(y, z) \in \langle 0, 4 \rangle \times \langle 6, 10 \rangle$ ,  $a = (2, 8)$ .



### Cvičení 15.92 na str. 129

Plocha  $z = \arccos(xy)$ ,  $(x, y) \in \langle -0.2, 0.9 \rangle \times \langle -1.1, 0 \rangle$ ,  $a = (\frac{1}{2}\sqrt{3}, -1)$ .



### Cvičení 15.93 a 15.94 na str. 129

Plocha  $y = \sin(x+z)/\sin(x-z)$ ,  $(x, z) \in \langle \frac{1}{4}\pi, \frac{3}{4}\pi \rangle \times \langle \frac{3}{4}\pi, \frac{5}{4}\pi \rangle$ ,  $a = (\frac{1}{2}\pi, \pi)$  a  
 $x = \lg(1+y^2+z^2) + \operatorname{arctg}(y+z)$ ,  $(y, z) \in (0, 1)^2$ ,  $a = (\frac{1}{2}, \frac{1}{2})$ .

