

[Soustava diferencialnich rovnic - Laplaceova transformace

[> **restart;**

[Prvni ze soustavy diferencialnich rovnic:

[> **dif_rov_1 := 5*x-(D(x))(t) +2*D(y)(t) + 10*y(t) = 0;**

$$dif_rov_1 := 5x - D(x)(t) + 2D(y)(t) + 10y(t) = 0$$

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[Druha ze soustavy diferencialnich rovnic:

[> **dif_rov_2 := (D(x))(t) +15*x(t) + 10*y(t) = 0;**

$$dif_rov_2 := D(x)(t) + 15x(t) + 10y(t) = 0$$

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[Laplaceova transformace prvni diferencialni rovnice.

[> **lap_1 := inttrans[laplace](dif_rov_1,t,s);**

lap_1 :=

$$\frac{5x}{s} - s \text{laplace}(x(t), t, s) + x(0) + 2s \text{laplace}(y(t), t, s) - 2y(0) + 10 \text{laplace}(y(t), t, s) = 0$$

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[Laplaceova transformace druhe diferencialni rovnice.

[> **lap_2 := inttrans[laplace](dif_rov_2,t,s);**

$$lap_2 := s \text{laplace}(x(t), t, s) - x(0) + 15 \text{laplace}(x(t), t, s) + 10 \text{laplace}(y(t), t, s) = 0$$

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[> **Obraz:=solve({lap_1,lap_2},{laplace(x(t),t,s),laplace(y(t),t,s)}**
);

$$Obraz := \{ \text{laplace}(x(t), t, s) = \frac{25x + 10x(0)s - 10y(0)s + x(0)s^2}{s(25s + s^2 + 75)},$$

$$\text{laplace}(y(t), t, s) = -\frac{1}{2} \frac{15x(0)s + 5sx - 2y(0)s^2 + 75x - 30y(0)s}{s(25s + s^2 + 75)} \}$$

[> **assign(Obraz):**

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[> **reseni_x := inttrans[invlaplace](laplace(x(t),t,s),s,t);**

$$reseni_x := \frac{x}{3} +$$

$$\frac{1}{39} \left(13 (3 x(0) - x) \cosh\left(\frac{5 t \sqrt{13}}{2}\right) - \sinh\left(\frac{5 t \sqrt{13}}{2}\right) \sqrt{13} (5 x + 3 x(0) + 12 y(0)) \right) e^{\left(-\frac{25 t}{2}\right)}$$

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> **reseni_y:=inttrans[invlaplace](laplace(y(t),t,s),s,t);**

$$reseni_y := -\frac{x}{2} +$$

$$\frac{1}{26} \left(13 \cosh\left(\frac{5 t \sqrt{13}}{2}\right) (2 y(0) + x) + \sqrt{13} \sinh\left(\frac{5 t \sqrt{13}}{2}\right) (3 x + 2 y(0) - 6 x(0)) \right) e^{\left(-\frac{25 t}{2}\right)}$$

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Po dosazeni pocatecnich podminek dostanu konkretni reseni : Napr. pro pocatecni podminky $x(0)=0$ a $y(0)=1$ obdrzim nize uvedene vysledky:

> **Pocatecni_podminky_x :=subs({x(0)=0,(y)(0)=1},reseni_x);**

Pocatecni_podminky_x :=

$$\frac{x}{3} + \frac{1}{39} \left(-13 x \cosh\left(\frac{5 t \sqrt{13}}{2}\right) - \sinh\left(\frac{5 t \sqrt{13}}{2}\right) \sqrt{13} (5 x + 12) \right) e^{\left(-\frac{25 t}{2}\right)}$$

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> **Pocatecni_podminky_y :=subs({x(0)=0,(y)(0)=1},reseni_y);**

Pocatecni_podminky_y :=

$$-\frac{x}{2} + \frac{1}{26} \left(13 \cosh\left(\frac{5 t \sqrt{13}}{2}\right) (2 + x) + \sqrt{13} \sinh\left(\frac{5 t \sqrt{13}}{2}\right) (3 x + 2) \right) e^{\left(-\frac{25 t}{2}\right)}$$

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